Prace Instytutu Mechaniki Górotworu PAN
Tom 15, nr 3-4, grudzień 2013, s. 131-146

Fractal structures and self-similar forms in the artwork of Salvador Dalí

Mariusz R. Sławomirski

Strata Mechanics Research Institute, Polish Academy of Sciences; Reymonta St., no. 27, 30-059 Krakow, Poland

Abstract

In the paper the author discusses fractal and self-similar forms encountered in various historic and modern branches of art. The idea of fractal forms and their properties have briefly been discussed. A number of 'non-typical' sets (Cantor set, Fig. 2, two- and three-dimensional Cantor dust, Figs. 3, 4, Sierpiński triangle, Fig. 10), semi-natural shapes (Barnsley fern, Fig. 1, lion male mane, Fig. 6), and natural shapes (silver fir, Fig. 5) have been presented as typical fractals shapes. The problem of the definition of fractal dimesion has been explained by means of the coastline length paradox. The intuitive idea of the Hausdorff dimension has been explained by means of covering of Britain coastline by the set of balls. Basing on the concept of the metric space understood as the pair consisting of the set and the metic attached to it, the exact definitions of the exterior measure, *s*-dimensional Hausdorff measure and Hausdorff dimension have been presented. The notion of the similarity dimension *D* has also been introduced in order to show a simple technique of determination of the fractal dimension. The author discusses fractal and self-similar shapes encountered in various branches of European, Islamic and Far East art. Referring to middle century art, the tracery in the window of the Cathedral of Milan (Fig. 11), and the façade of the Church of the Trinity, Vendôme, France (Fig. 12) have been discussed as examples of fractal and self-similar forms in the flamboyant Gothic. The interiors of Alhambra (Fig. 13) and Taj Mahal (Fig. 14) are depicted as examples of fractal forms in the islamic art. *The Great Wave off Kanagawa* painted by Katsushika Hokusai about 1830 and shown in Fig. 15 is the example of fractal structures present in the Japanese art. Fractal, self-similar and curvilinear motives typical for Art Nouveau are discussed and presented in sect. 3 (Figs. 16-18 and 104). The author presents the detailed analysis of two artworks of Salvador Dalí: *The Persistence of Memory* and *The Burning Giraffe*. The similarity dimension of the fragmented table contained in *The Disintegration of The Persistence of Memory* is eveluated. The discussion of a number of self-similar motives in The *Burning Giraffe* has also been presented. The purpose of the paper is to demonstrate that fractal and self-similar motives may be encountered in various branches of European and non-European art developed in various historic periods.

Keywords: fractals, self-similar forms, fractal dimension, flamboyant Gothic, Art Nouveau, surrealism art, Salvador Dalí, The Persistence of Memory, The Burning Giraffe

1. Fractals

Fractals are 'rough' geometric sets that are self-similar in the exact, stochastic or at least approximate manner. The self-similarity denotes that the enlarged defined piece represents the same pattern as the entire structure. The term *fractal* understood in the sense outlined above was used the first time by Benoît Manderlbrot in 1975. Several years after publication of his book *The Fractal Geometry of Nature* (1982) fractals were comprehensively known in the scientific society.

From the mathematical standpoint the concept of fractal is difficult to be defined even for mathematicians. Consequently, any exact definition of fractal has not commonly been accepted up to the present time. On the other hand, at present time the notion of fractal is well-known, and fractals are usually regarded as 'rough' sets possessing the following features:

i. they are non-trivially self-similar in the exact, stochastic or at least approximate manner,
ii. their form is too rough and too elaborate to be described by means of traditional geometry,

iii. they are continuous but 'nowhere differentiable', and consequently they cannot be measured in traditional ways,
iv. their Hausdorff dimension is greater than their topologic dimension,
v. they possess simple recursive definitions.

The example of a fractal pattern is the Barnsley fern prexented in Fig. 1. It may easily be seen that enlarged one leaf of the fern represents the same pattern as the entire structure. Moreover, the fern image obtained by means of recursive computer operations satisfies fractal features i.-v. mentioned above.

Another example of the fractal structure is represented by the well-known Cantor set. It is created in a very simple manner. A section of a straight line is divided into three equal parts, and the central part is removed. The procedure is repeated many times with respect to non-removed subsections as it has been shown in Fig. 2.

Fig. 1. The Barnsley fern. From: http://underdown.wordpress.com/category/physics/

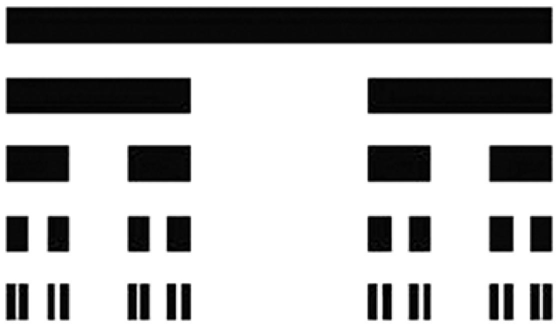

Fig. 2. The Cantor set. From: http://earsketch.gatech.edu/learning/self-similarity-and-recursion-4-the-cantor-set

The two-dimensional homologue od the Cantor set is the so-called Cantor square or two-dimensional Cantor dust. It is created in the mode similar to the Cantor set as it has been depicted in Fig. 3.

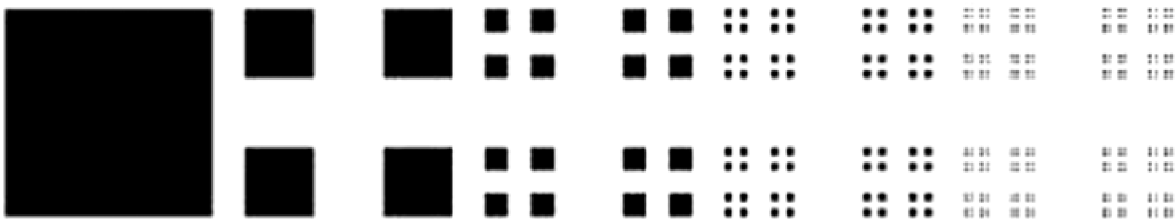

Fig. 3. Cantor squares. From: http://mathworld.wolfram.com/CantorDust.html

The three-dimensional homologue of the Cantor set constitutes the so-called three-dimensional Cantor dust or briefly Cantor dust. It is created in the mode similar to the Cantor set as it has been shown in Fig. 4.

Fractal structures are commonly encountered in biology, and they represent geometric forms of many living organism. The examples are shown is Fig. 5 and 6. Fig. 5 depicts the silver fir the population of which is very high in the Carpathian Mountains. Fig. 6 presents the lion's head with a spectacular mane.

The computer algorithms based on recursive procedures enable the specialists to create elaborate graphical patterns of biological forms, as flowers, trees, and even animals. Fig. 6 presents the image of

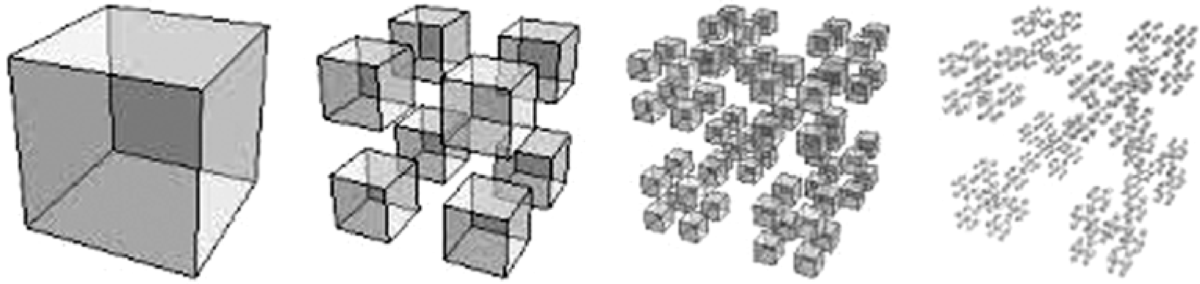

Fig. 4. The 3D Cantor dust. From: http://www.robertdickau.com/cantor.html

Fig. 5. The self-similar form of a silver fir encountered commonly in the Carpathian Mountains. From: http://drzewa.nk4.netmark.pl/atlas/jodla/jodla_pospolita/jodla_pospolita.php

Fig. 6. The example of the fractal art: The computer created lion male head with a spectacular mane. From: http://fineartamerica.com/featured/1-male-lion-fractal-julie-l-hoddinott.html

a lion male mane obtained by means of computer simulation. At present time artificial patterns of animals and other biological forms are applied comprehensively for modern movie production.

2. Fractal dimension

The evaluation of fractal roughness constituted a serious problem. Since fractal cannot be measured in traditional manner the technique of determination of fractal dimension must differ from that encountered in differential geometry. An approach to apply the traditional measuring techniques leads often to non-unique solutions.

If anybody tries to measure the length of the Britain coastline by means of sticks, the final result is dependent strongly on the length of sticks. Applying smaller and smaller sticks one obtains greater and greater magnitude of overall coastlength as it has been shown in Fig. 7.

Consequently, the ‘classical’ measurement technique applied here is completely unsuccessful, and it leads to the so-called coastline paradox (Mandelbrot, 1967). It is obvious that for irregular and rough forms another measuring technique must be applied. The fractal dimesion must directly be related to its complexity and to its space filling capacity.

A new concept of measurement of rough shape sets was formulated one hundred years ago by Felix Hausdorff (1919). His idea consists of ‘covering’ of rough boundaries by sequences of balls.

Fig. 7. Unsuccessful traditional technique of determination of the length of Britain coastline. 11.5 sticks of length 200 km gives 2300 km (left side), 28 sticks of length 100 km gives 2800 km (centre), and 70 sticks of length 50 km gives 3500 km (right side)

The cover of the Britain coastline by balls of various radii is presented in Fig. 8. It may be seen that the precision of estimation of the coastline is increased progressively when the radii of balls r are reduced and the number of balls N is increased. Speaking otherwise, for small r the magnitude of N is great and vice versa. For appointed r the number of balls N to cover a rough shape is unique, i.e. there exists a function $N(r)$ for the defined rough shape.

Fig. 8. The idea of the Hausdorff dimension for the Britain coastline: The covering of the coastline by balls. From: http://en.wikipedia.org/wiki/Hausdorff_dimension

According to the idea of Hausdorff N is proportional to $1/r^D$, i.e.:

$$N \propto \frac{1}{r^D} \tag{1}$$

where D is the critical number for the defined rough set. If d is greater than D then the number of balls $N = 1/r^d$ is insufficient to cover the set. If d is less than D then the number of balls $N = 1/r^d$ will be ovrabundant. The critical number D for the defined rough set is called its Hausdorff dimension.

The exact mathematical definition of the Hausdorff dimension is related to the theory of metric spaces, and it is much more elaborate in comparison to intuitive notion presented above.

For the defined set $\boldsymbol{Y}$ one may often introduce the metric, i.e. the measure of distance $\rho(u,w)$ between each pair of elements u and v of $\boldsymbol{Y}$. If u,v,w are elements of $\boldsymbol{Y}$, the metric must satisfy the following conditions:

$$\rho(u,w) \in R^{+} \qquad \text{(the metric is non-negative)} \tag{2}$$

$$\rho(u,w) = 0 \Leftrightarrow u = w \qquad \text{(identity of indiscernibles)} \tag{3}$$

$$\rho(u,w) = \rho(w,u) \qquad \text{(symmetry)} \tag{4}$$

$$\rho(u,w) \le \rho(u,v) + \rho(v,w) \qquad \text{(triangle inequality)} \tag{5}$$

An ordered pair $(\boldsymbol{Y},\rho)$ consisting of the set $\boldsymbol{Y}$ and the metric ρ on it is called the metric space.

The example of the metric space is three dimensional Euclidean space for which the distance between points A and B is defined by:

$$\rho(A,B) \overset{\text{df}}{=} \sqrt{\left(A_x - B_x\right)^2 + \left(A_y - B_y\right)^2 + \left(A_z - B_z\right)^2} \tag{6}$$

where A_x, A_x, A_z respectively denote x, y, and z Cartesian coordinates of point A, and B_x, B_x, B_z respectively denote x, y and z Cartesian coordinates of point B. It may easily be checked that the Euclidean metric satisfies conditions (2)-(5).

The taxi metric called also the Manhatan metric shown in Fig. 9 is defined by:

$$\rho(A,B) \overset{\text{df}}{=} \left|A_x - B_x\right| + \left|A_y - B_y\right| \tag{7}$$

The Manhatan metric expresses the length of way which must be passed by a taxi to transfer from point A to point B according to the system of orthogonal streets. It may easily be checked that the taxi metric satisfies also conditions (2)-(5).

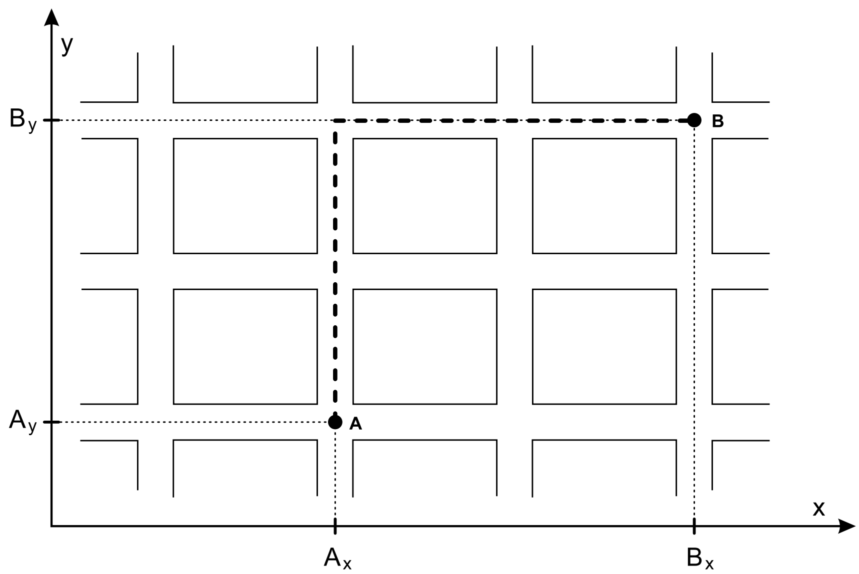

Fig. 9. The system of orthogonal streets and the taxi metric

The Manhatan metric may directly be generalised to the state road system. Note, that the Manhatan metric space shown in Fig. 9 and its generalisations are the examples of non-Euclidean spaces.

In the metric space $(\boldsymbol{Y}, \rho)$ the ball $K_{c,r}$ of radius r and centred at a point c is defined as the subset of all elements of $\boldsymbol{Y}$ distant not more than r from the centre c, i.e.:

$$K_{c,r} \overset{\text{df}}{=} \left\{u : u \in \boldsymbol{X} \ \text{ and } \ \rho(u,c) \le r\right\} \tag{8}$$

The exterior measure $H_\delta^s(\boldsymbol{V})$ of the subset $\boldsymbol{V}$ of $\boldsymbol{Y}$ is the lower limit (infimium) of the sum of all radii r (not exceeding δ, and taken in s power) of balls which enable us to cover the subset $\boldsymbol{V}$, i.e.:

$$H_\delta^s(\boldsymbol{V}) \overset{\mathrm{df}}{=} \inf\left\{\sum_{i=1}^{n} r_i^s : \boldsymbol{V} \text{ is covered by balls of radii } 0 \le r_i \le \delta\right\} \tag{9}$$

When δ is reduced then the exterior measure is increased progressively. Consequently, one may tend to the limit when $\delta \to 0$.

The s-dimensional Hausdorff measure $H^s(\boldsymbol{V})$ of $\boldsymbol{V}$ subset is defined as the limit of the exterior measure $H_\delta^s(\boldsymbol{V})$ when $\delta \to 0$:

$$H^s(\boldsymbol{V}) \overset{\mathrm{df}}{=} \lim_{\delta\to 0}\left\{H_\delta^s(\boldsymbol{V})\right\} \tag{10}$$

The Hausdorff dimension of $\boldsymbol{Y}$ set is defined as the lower limit (infimium) of Hausdorff measures $H^s(\boldsymbol{Y})$ equal to zero:

$$\dim_H(\boldsymbol{Y}) \overset{\mathrm{df}}{=} \inf\left\{ s \ge 0 : H^s(\boldsymbol{Y}) = 0\right\} \tag{11}$$

The Haussdorf dimension may be regarded as the generalization of the topological dimension. The Hausdorff dimension of a straight line is equal to 1, the Hausdorff dimension of a plane is equal to 2, etc. On the other hand, for rough sets as fractals their Hausdorff dimensions may be rational or even non-rational positive numbers. The Hausdorff dimension represents then the measure of ‘space covering’, and its magnitude depends strongly on the roughness of the set.

The determination of Hausdorff measures of rough sets based on the formal definition presented above is not easy. Fortunately, for the majority of fractals their Hausdorff dimension is equal to the self-similarity dimension D the definition of which is relatively simple.

In the first stage of creation of the Cantor set presented in Fig. 2 the length of the initial line segment is reduced three times constituting the ‘construction element’. The recurence operation consists of location of two constructions elements in one line in such a manner that the ‘empty’ distance between them is equal to the length of the construction element. Taking into account the analogy with balls coverting the Britain coastline (Fig. 8, Eq. (1)) one may assume that the number of elements used for construction N is proportional to the scale of reduction of initial segment σ taken in D-th power, i.e.

$$N \propto \sigma^D \tag{12}$$

Assuming the proportionality coefficient equal to one, we have:

$$N = \sigma^D \tag{13}$$

Basing on Eq. (13) the similarity dimension D may easily be obtained in the following from:

$$D = \frac{\ln N}{\ln \sigma} \tag{14}$$

For the Cantor set presented in Fig. 2 we have triple initial size reduction, i.e. $\sigma = 3$, and two building elements, i.e. $N = 2$, and consequently for the considered case the similarity dimension is:

$$D = \frac{\ln 2}{\ln 3} \cong 0.6309 \tag{15}$$

The proof that the Hausdorff dimesion of the Cantor set is identical with its similarity dimension, i.e. it is equal to ln2/ln3 may be found in a number of mathematical papers, e.g. http://www.mathdb.org/notes_download/advanced/fractals/e_dim_H.pdf.

For the Sierpinski triangle shown in Fig. 10 we have double initial size reduction, i.e. $\sigma = 2$, and three construction elements, i.e. $N = 3$, and consequently for the considered case the similarity dimension is:

$$D = \frac{\ln 3}{\ln 2} \cong 1.585 \tag{16}$$

Fig. 10. The recursive procedure of creation of the Sierpiński triangle.
From: https://orderinchoas.files.wordpress.com/2013/05/sierpinskytrianglewikipedia.gif

It may be proved that the Hausdorff dimesion of the Sierpiński triangle is identical with its similarity dimension, i.e. it is equal to ln2/ln3. The compatibility of the Hausdorff dimesion and the similarity dimension occurs for almost all fractals. Rare exceptions from this rule have been noted in the scientific literature.

3. Self-similar and fractal patterns in the art

The self-similar and fractal pattern may be encountered in various branches of old and modern art.

The Gothic architecture involves many examples of fractal-type forms. Is refers especially to the late Gothic floric style forms constituting the so-called flamboyant Gothic pattern. Fractal-type forms are typical for the tracery, i.e. the stonework elements that support the glass at the top zone of a Gothic window. Fig. 11 presents the tracery in the window of the Cathedral of Milan, Italy.

Fig. 11. Self-similar pattern of the tracery in the window of the Cathedral of Milan, Italy. From: http://www.bluffton.edu/~sullivanm/italy/milan/cathedral/cath2.html

Fig. 12 presents the self-similar pattern of the façade of the Church of the Trinity erected in the flamboyant Gothic style and located in Vendôme, France.

Islam prohibited to present the views of people and animals in the architecture, especially in sacral buildings. Consequenltly, islamic artists developed highly elaborate decorative forms. They are often based on self-similar and fractal shapes. The example is the interior of Alhambra, a royal palace erected in the Magreb islamic style in Grenada, Andalusia, Spain (Fig. 13).

Fig. 12. Fractal structure of the façade of the Church of the Trinity, Vendôme, France. From: http://en.wikipedia.org/wiki/Flamboyant

In the architecture the islamic artists applied often elaborate fractal shapes based on floric motives. Presentation of plants was not prohibited by a raw arabic morality, and floric forms represent one of the most important elements of islamic decoration art. Fig. 14 presents fractal-type, floral based ornaments in the interior of Taj Mahal in Agra, India. Fractal elements are also visible explicitly in the historic arabic caligraphy.

Fractal pattern is explicity visible in the Far East artwork. Fig. 15 presents the *Great Wave off Kanagawa*, a famous work by Katsushika Hokusai, a Japanese artist who was born in 1760 and died in 1849. According to the information presented by Radio UNAM Edmond de Goncourt, the French writer and art critic, described the artwork considered here in the following manner: "The drawing of the wave is a deification of the sea made by a painter who lived with the religious terror of the overwhelming ocean completely

Fig. 13. The example self-similar shapes in the islamic architecture: the interior of Alhambra, Granada, Spain. From: http://trendwallpaper.com/alhambra-architecture-wallpaper-download-hd-photo-background.html

Fig. 14. Self-similar ornaments in the interior of Taj Mahal, Agra, India. From: http://www.taj-mahal.net/augEng/textMM/dodosengN.htm

surrounding his country; He is impressed by the sudden fury of the ocean's leap toward the sky, by the deep blue of the inner side of the curve, by the splash of its claw-like crest as it sprays forth droplets".

The flamboyant Gothic and Japanese art were inspiration for the formulation of so-called Art Nouveau, a new style of decorative and fine art developed in the last decade of 19th century and the first decade of 20th century. Art Nouveau was the reaction to the academic and historic styles of 19th century art based on classical and ancients forms.

Fig. 15. A fractal pattern in *The Great Wave off Kanagawa* by Katsushika Hokusai (about 1830). From: http://en.wikipedia.org/wiki/File:The_Great_Wave_off_Kanagawa.jpg

Fig. 16. The example of fractal pattern in the Art Nouveau graphics. From: http://typographya.wordpress.com/2012/02/26/fuentes-art-nouveau-fuentes-art-deco/

Fig. 17. Self-similar forms in the Art Nouveau style graphics Zioła (The Herbs) by Stanisław Wyspiański. From: http://www.tatento.pl/blog/2013/06/19/ziola-wyspianskiego-22-czerwca-w-rydlowce/

Fig. 18. Art Nouveau style Woman with gourgeous hair. From: http://pinnest.net/nouveau-style-woman-with-gourgeous-hair/

Art Nouveau embraced architecture, graphics, interior design, various forms of utilitar and decorative art, furniture, textiles, as well as fine art. The characteristic feature of Art Nouveau was the replacement of straight elements typical for ancient forms by soft, smooth, curvilinear shapes, and introduction of flame- and shell-like textures. In the Art Nouveau the absence of visual perspective and chirocuro was counterbalanced by the abundance of curved lines and elaborate ornaments. The Art Nouveau artists applied highly stylised natural, organic and floristic forms as a source of inspiration to obtain the stylistic harmony of the artwork.

Since the artist applied commonly floristic decorative elements, and natural organic and plant shepes are often self-similar, the fractal-like forms may be rearded to be embedded in essence of the Art Nouveau. It is visible explicitly in the vault of the Sagrada Familia Church in Barcelona, one of most spectacular object of NeoGothic and Art Nouveau architecture designed by Antoní Gaudi (Fig. 104, separate page).

The beginning of the World War I is regarded as the end of Art Nouveau style. In twenteenth and thirteenth years of 20th century the Art Nouveau was rejected, and it was even regarded as a form of a 'wrong taste'.

4. The Persistence of the Memory

One of the first modern artist who expressed his admiration with respect to Art Nouveau was Salvador Dalí, a spectacular Spanish Catalan painter regarded often as one of the most prominent artists of 20th century. In his paintings and sculptures one may study often soft, wave-pattern lines, inspired undoubtly by the Art Nouveau pattern.

On the other hand, owing to his great artistic creativity the art of Salvador Dalí is highly elaborate, and it contains the wealth of rational as well as irrational elements. The activity of Dalí is directly related to surrealistic art which assumes the "fundamental crisis of object".

Possibly, such a standpoint was inspired by the phenomenological philosophy initiated and developed in the beginning of 20th century by German philosopher Edmund Husserl. According to Husserl's concept (1913, 1925, 1923, 1928, 1936) the objects are presented in our consciousness in the form of phenomena, which may be regarded as intentional, irreal corelates of pure consciousness. The objects are created by the conscousness in a highly elaborate and multi-level constitution process from the so-called hyletic data, i.e. from the most primitive impressions which do not represent any really existing thing.

From point of view represented by surrealists the crisis of object consist of the situation in which an object is being thought as the superposition of a fixed and really existing external object as well as the extetension of our subjective self. Consequently, the objects presented by surrealists are represented often by the so-called phantom objects.

The main idea of Dalí consists of the linking of objects separated always in the real world by the 'physical distance' as well as in the 'denotation' manner. The linked objects are deformed by the artist in a highly subjective mode, similar to that encountered in dreams or deliric states. The idea outlined above has been developed by Dalí into the so-called paranoiac-critical method. The considered method enables the brain of the artist to perceive links between things which are never linked rationally in a real world. According to Dalí the paranoiac-critical method represents "*a spontaneous method of irrational knowledge based on the critical and systematic objectivity of the associations and interpretations of delirious phenomena*". Consequently, the objects presented by the artist have a "minimum of mechanical meaning, but when viewed the mind evokes phantom images which are the result of unconscious acts".

In 1931 Dalí painted his one of most famous works, *The Persistence of Memory*, in Spanish *La persistencia de la memoria* (Fig. 101, on a separate page). The artwork involves the realistic painting technique, visual perpective, chiaroscuro, as well as absurdal confrontion of various objects and their properties. The most characteristic feature of the work is the so-called onirism, i.e. an aristic convention (applied e.g. in 20th century literature) consisting of the presentation of reality in the form of a dream, sometimes amiorable, sometimes nighmarish, with absurdal contrasts, irrational transitions, and sometimes violation of time sequence as well as violation of the sequence of reason and outcome.

The colouring of the Dalí's artwork is balanced. The author aviods to use aggressive composition of colors. Variations of white, blue, bronze, gold and black cover the majority of the area of painting.

In The Persistence of Memory various objects taken from the real world are presented in non-conventional manner. Objects are located in a special mode to make the impression of large space and emptiness. The sensation of profoundness has been obtained by the background painted in the form of marine coast landscape with steep rocks illuminated by sunbeams at the sunset. The specialists explained that the realistic image of coast represents Cap de Creus peninsula near which Dalí and his wife Gala spent often their summer holidays.

All objects presented in the picture may be divided into 'hard' and 'soft'. It reflects Dalí's theory of 'hardness' and 'softness' which was primary for his thinking at the time. The hard objects are represented by coastline rocks mentioned above and by brown table located at the left side of the picture. Soft elements are represented by three melting pocket watches. The fourth, closed watch of the gold colour is located at the table near lower and left corner of the picture. The ants covering and attacking the watch may ofted be encountered in the artwork of Dalí. They represent the disintegration and decay of the accompanying object. In the considered case the ants attack the time control system respresented by gold watch. The scene symbolizes the lapse of human existence and the decay of human memory ordered and sequenced by time defined and measured by the watch.

Three melting watches represent soft elements dispatched at the picture. Their softness is the symbol of the relativity of time. Dalí was highly interested in the ideas of modern physics, and the Einstein's special theory of relativity which stated the dependence of time on the observer. It is possible that the theory of relativity has been the inspiration for the Persistence of the Memory. According to Dawn Ades "The soft watches are an unconscious symbol of the relativity of space and time, a surrealist meditation on the collapse of our notions of a fixed cosmic order".

It is possible that the Husserl's concept of the phenomenology of inner time consciousness (cf. Husserl, 1928) was one of significant factors for the inspiration of the problem of subjective perception of time involved by Dalí in his artwork.

On the other hand, many experts are of opinion that the genuine inspiration for creeping watches was much more simple. The image of melting watches was inspired by artist's considerations about the softness of a camembert chease which was eaten by him for the lunch. Dalí painted alreday unfinished picture with the carefuly prepared background, but he had not idea what to present at the center. The softness of the camambert chease molten in the sun inspited him to nest creeping watches at the artwork.

One of molten clocks is stretched on the piece of human figure located in the centre of the composition. The deformed human face with explicit nose and closed eyes symbolizes probably the self-portrait of the artist.

In 1954 Dalí returned to the theme of molten watches, and painted a new version of the picture, entitled *The Disintegration of the Persistence of Memory.* (Fig. 102, separate page). It may be seen that the hard elements except those in the background, i.e. the table and the bottom of the composition have been fragemented into smaller, repeating elements. Consequently, the table constituting hard element has been presented in the form of repeating rectangular blocks.

It is easy to see that the blocks represent the set similar to the Cantor dust. For the 3-dimensional Cantor dust presented in Fig. 4 we have $N = 8$, and $\sigma = 3$, and consequently its fractal dimension D is:

$$D = \frac{\ln 8}{\ln 3} \cong 1.8928 \tag{17}$$

In the case of 2-dimensional Cantor dust presented in Fig. 3 have $N = 4$, and $\sigma = 3$, and consequently its fractal dimension D is:

$$D = \frac{\ln 4}{\ln 3} \cong 1.2619 \tag{18}$$

For the plane of fragmented table in the picture we have $\sigma = 12 + 11/5 = 14.2$ (mesured) and $N = 12 \times 12 = 144$ if we regard blocks as topologically two-dimensional. Consequently, the similarity dimension for the fragmented plane of the table is:

$$D = \frac{\ln 144}{\ln 14.2} \cong 1.8731 \tag{19}$$

For the vertical side of the table one may obtain another magnitude of the similarity dimension.

It is possible that the fragmentation of the table into blocks constitutes the symbol of atomic structure of realistic elements of the picture whereas non-realistic terms as melting watches remained continuous because they are 'non-physical' and they represent the imagination produced by our consciousness. Moreover, the fragmentation mentioned above reveals further imagery through the gaps between blocks presenting something located beneath the original work.

5. The Burning Giraffe

Another famous artwork by Salvador Dalí, *The Buring Giraffe*, in Spanish *La jirafa ardiente* was painted by him in 1937. The artwork depicted in Fig. 102 on a separate sheet presents the apocaliptic vision of human existence, oniric image of human fate and fascination of death as the final attractor of human life.

At the foreground we see irrational, extremely slim, anorectic, and helically deflected female shape, the Venus without face. The female stays within raw, waste land type landscape which implies the impression of lifelessness and emptiness referring to the entire picture. The emptiness-type landscape is illuminated by blue light in such a manner that illuminating light cannot precisely be distinguished from the blue colour of the sky. The author obtained in that mode the sensation of deeps and hugeness of the landscape. The bystander is under the impression that the human and animal-like shapes embedded in the picture have been nested in the emptiness against their intention and volition.

The female body, in certain parts in the decay state, is surrounded by a tiny textil material. Her hands, with narrow but developed muscles deprived of skin are looking for the support in the space. She probably intends unsuccessfully to find the contact with the real world located possibly outside, and to get out the emptiness zone.

The legs of Venus without face are in motion. The impresion of their movement has been obtained by means of the wave-like forms of deformation of the dress. Their similarity and approximate self-similarity is directly visible. The author of this paper is of opinion that the artist intended to present the motion by means of tiny, fractal-like forms.

The wave-like shapes represent outdoubly the reminiscence of the Art Nouveau ornaments which were the object of fascination of Dalí. It is not a singular case that he sometimes is described as surrealistic flamboyant artist.

Self-similar shapes may also be found in the construction of hand muscles of Venus not covered by the skin.

The body of Venus deprived of face constitutes the chest of semi-opened empty drawers. They represent probably the symbol of most dark aspects human sub-consciousness: phobies, hidden desires, primitive instincts, secrets, and drawers that may only be opened through psychoanalysis.

Venus deprived of face represents the symbol of human isolation and infirmity as well as inability to conquer the fate imposed independently of the personal intention and activity. Such an interpretation involves the existential concept of human life developed by Martin Heidegger (1927).

In the right side of the picture we see another very slim female shape surrounded by tiny textil material. Her face is non visible, and she may be regarded as the second Venus deprived of face. The top of her head constitutes the support for a leafless, dead tree. The branches of the tree constitute tiny, explicitly self-similar forms. In right hand of the woman we see a long stripe of meat. It probaly means that in the emptiness space presented in the picture the human body is regarded as a peace of meat and nothing else.

The bodies of both women are equiped with undefined, fallic-like forms protruding from woman's backs. The forms are supported by crutch-like objects. Such a construction consisting of fallic forms supported by crutch reflect erotic fantasies of Dalí and his considerations referring to the role of sexuality in human existence. They may also be found in many other paintings of the artist.

The burning giraffe has been located in the background, in the left side of the artwork. Although the giraffe has been shown applying the quasi-realistic shape, the flames are presented using highly twisted self-similar forms. The flames represent the succeeing zones in the picture in which the self-similarity and the reminiscence of Art Nouveau have been applied by the author in his artistic concept.

There is a number of interpretations of the symbolism of giraffe presented in the picture. According to the author of this paper the giraffe represents the destruction of human civilisation represented by the 'classic', realistic forms of the animal. The destruction of civilisation creates the emptiness in which human existence represented by Venuses without faces is deprived of any sense, and any support.

The reader must be aware that Dalí painted The Burning Giraffe at the time of Spanish domestic war. The both sides, comunists and nationalists, based on antagonistic ideologies continued ruthless war tending to complete extermination of opponents.

It is possible that the destruction of giraffe as well as the entire picture represent the symbol of the disater of civilisation implied by the war excitated by antagonistic ideologies. Females without faces presented in the figure represent the fate of ordinary people not involved in political or ideologic confrontations. At the war conditions their future does not depend on their activity. For them there is not an alternative, escape or recourse from the fate determined by the military conditions.

6. Recapitulation

A brief review presented above proves that independently of the content of artworks the fractal and self-similar motives may be encountered in various branches of European and non-European art developed in various historic periods, involving modern, surrealistic zone.

References

Barnsley M.F., Rising H., (1993): *Fractals Everywhere*. Boston: Academic Press Professional.

Edgar G., (2008): *Measure, topology, and fractal geometry* , Springer-Verlag, New York.

Falconer K., (1997): *Techniques in Fractal Geometry*, John Wiley & Sons.

Falconer K., (2003): *Fractal Geometry: Mathematical Foundations and Applications*, John Wiley & Sons.

Gordon N., (2000): *Introducing fractal geometry*, Icon, Duxford.

Gouyet J.F., (1996): *Physics and fractal structures*, Masson Springer, Paris - New York.

Hausdorff F., (1919): *Dimension und äußeres Maß*, Mathematische Annalen, **79**, 157.

Heidegger M., (1927): *Sein und Zeit*, Niemeyer, Tübingen.

Hurewicz W., Walkman H., (1948): *Dimension Theory*, Princeton University Press.

Husserl E., (1913): *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie. Erstes Buch: Allgemeine Einführung in die reine Phänomenologie*, Niemeyer, Halle.

Husserl E.,(1930): *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie. Zweites Buch: Phänomenologische Untersuchungen zur Konstitution*, (manuscripts), published as: *Husserliana*, Bd. 4, Nijhoff, The Hague, (1952).

Husserl E., (1930): *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie. Drittes Buch: Die Phänomenologie und die Fundamente der Wissenschaften*, (manuscripts), published as: *Husserliana*, Bd. 5, Nijhoff, The Hague, (1971).

Husserl E., (1925): *Erste Philosophie. Erster Teil: Kritische Ideengeschichte*, (manuscripts), published as: *Husserliana*, Bd. 7, Nijhoff, The Hague, (1959).

Husserl E., (1923 - 1924): *Erste Philosophie. Zweiter Teil: Theorie der phänomenologischen Reduktion*, (manuscripts), published as: *Husserliana*, Bd. 8, Nijhoff, The Hague, (1959).

Husserl E., (1918 - 1926): *Analysen zur passiven Synthesis*, (manuscripts), published as: *Husserliana*, Bd. 11, Nijhoff, The Hague, (1966).

Husserl E., (1928): *Vorlesungen zur Phänomenologie des inneren Zeitbewusstseins*, Niemeyer, Halle.

Husserl E., (1931): *Cartesianische Meditationen und Pariser Vorträge*, (manuscripts), published as: *Husserliana*, Bd. 1, Nijhoff, The Hague, (1973).

Husserl E., (1936): *Die Krisis der europäischen Wissenschaften und die transzendentale Phänomenologie: Eine Einleitung in die phänomenologische Philosophie*, Philosophia, Belgrad, 1, 77 – 176, reprinted as: *Husserliana*, Bd. 6, Nijhoff, The Hague, (1976).

Husserliana (1952 –): *Edmund Husserl Gessamelte Werke*, Bd. 1 – 42, Husserl Archiv te Leuven, Leuven, Belgium.

Mandelbrot B.B., (1967): *How Long Is the Coast of Britain? Statistical Self-Similarity and Fractional Dimension*, Science, New Series, Vol. 156, No. 3775.

Mandelbrot B.B., (1982): *The Fractal Geometry of Nature*, Freeman, San Francisco.

Mandelbrot B.B., (2004): *Fractals and Chaos*, Springer, Berlin.

Michalski E., (1925): *Die entwicklungsgeschichteliche Bedeutung des Jugendstils*.

Peters E., (1996): *Chaos and order in the capital markets: a new view of cycles, prices, and market volatility*, John Wiley & Sons, New York.

Radio UNAM, (2010): *La Gran Ola de Kaganawa* [in Spanish], Retrieved 7 July 2010.

Taylor R., Micolich A.P., Jonas D.: *Fractal Expressionism: Can Science Be Used to Further Our Understanding of Art?*, Faculty of Scence, University of New South Wales, Australia.

Wikipedia: http://en.wikipedia.org/wiki/Alhambra

Wikipedia: http://en.wikipedia.org/wiki/Art_Nouveau.
Wikipedia: http://en.wikipedia.org/wiki/Coastline_paradox .
Wikipedia: http://en.wikipedia.org/wiki/Flamboyant .
Wikipedia: http://en.wikipedia.org/wiki/Fractal .
Wikipedia: http://en.wikipedia.org/wiki/Hausdorff_dimension .
Wikipedia: http://en.wikipedia.org/wiki/How_Long_Is_the_Coast_of_Britain%3F_Statistical_Self-Similarity_and_Fractional_Dimension .
Wikipedia: http://en.wikipedia.org/wiki/Islamic_art .
Wikipedia: http://en.wikipedia.org/wiki/Taj_Mahal .

Struktury fraktalne i kształty samopodobne w sztuce Salvadora Dalí

Streszczenie

W artykule autor omawia formy fraktalne i samopodobne spotykane w rozmaitych kierunkach sztuki, zarówno historycznych jak i współczesnych. Krótko przedstawiona została idea fraktali i form fraktalnych jako nietypowych tworów geometrycznych o specjalnych własnościach matematycznych. Jako przykłady form fraktalnych pokazano jedno- dwu- i trój-wymiarowe zbiory Cantora (Fig. 2, 3, 4), trójkąt Sierpińskiego (Fig. 10), kształty pół-naturalne, jak paproć Barnsleya (Fig. 1), lwią grzywę (Fig. 6), a także formy naturalne jak jodłę karpacką (Fig. 5). Trudności ze zdefiniowaniem wymiaru fraktala w tradycyjnym sensie pokazano na przykładzie paradoksu pomiaru długości wybrzeża. Ideę wymiaru Hausdorffa przedstawiono w sposób intuicyjny poprzez pokrycie wybrzeża Wielkiej Brytanii zbiorem kul. Wychodząc z koncepcji przestrzeni metrycznej rozumianej jako para złożona ze zbioru i rozciągniętej na nim metryki rozumianej w sensie topologicznym, podano ścisłe definicje miary zewnętrznej, miary Hausdorffa, wymiaru Hausdorffa i wymiaru faktalnego. Zdefiniowano również wymiar podobieństwa (niemal zawsze równy wymiarowi Hausdorffa) jako prostą metodę wyznaczania wymiaru fraktalnego. Po przedstawieniu pojęć wstępnych autor omawia formy fraktalne i kształty samopodobne spotykane w rozmaitych kierunkach sztuki europejskiej i pozaeuropejskiej. Odnosząc się do architektury średniowiecza jako przykłady form fraktalnych występujących w gotyku płomienistym przedstawiono maswerk w oknie Katedry w Mediolanie (Fig. 11) oraz fasadę Kościoła pod wezwaniem Św. Trójcy w Vendôme we Francji (Fig. 12). Fragmenty wnętrza Alhambry w Grenadzie (Fig. 13) oraz mauzoleum Taj Mahal w Agrze (Fig. 14) pokazano jako przykłady fraktalnych form w sztuce islamu. Wielka Fala w Kaganawie namalowana przez japońskiego artystę Katsushika Hokusai około 1830 roku i pokazana na Fig. 15 stanowi przykład form fraktalnych stosowanych w sztuce Dalekiego Wschodu. Fraktalne i samopodobne kształty wbudowane w systemy wymyślnie powikłanych linii krzywych typowych dla sztuki secesji omówiono w rozdziale 3, uzupełnionym Fig. 16-18 oraz Fig. 104. W następnych dwóch rozdziałach autor analizuje szczegółowo dwa słynne dzieła Salvadora Dalí: *Trwałość Pamięci oraz Płonącą Żyrafę*. Oszacowano wymiar fraktalny rozbitego w pył Cantora stołu występującego w obrazie *Rozkład Trwałości Pamięci* pochodzącego z 1952 roku i stanowiącego zmodyfikowaną wersję *Trwałości Pamięci* z 1931 roku. Omówiono elementy fraktalne i samopodobne występujące w obrazie Płonąca Żyrafa. Podano też interpretację zawartości obydwu obrazów. Zasadniczym celem artykułu było pokazanie, że motywy fraktalne i samopodbne występowały i występują nadal w rozmaitych kierunkach sztuki zarówno europejskiej jak i pozaeuropejskiej odnoszących się do różnych epok historycznych.

Słowa kluczowe: fraktale, kształty samopodobne, wymiar fraktalny, gotyk płomienisty, sztuka islamu, secesja, surrealizm, Salvador Dalí, *Trwałość Pamięci*, *Płonąca Żyrafa*

Fig. 101. *The Persistence of Memory* by Salvador Dalí (1931).
From: http://www.wikipaintings.org/en /salvador-dali/the-persistence-of-memory-1931

Fig. 102. *The Disintegration of the Persistence of Memory* by Salvador Dalí (1952).
From: http://www.wikipaintings.org/en/salvador-dali/the-disintegration-of-the-persistence-of-memory

Fig. 103. *The Burning Giraffe* by Salvador Dalí (1952). From: http://postcardsworldwide.wordpress.com /tag/salvador-dali/

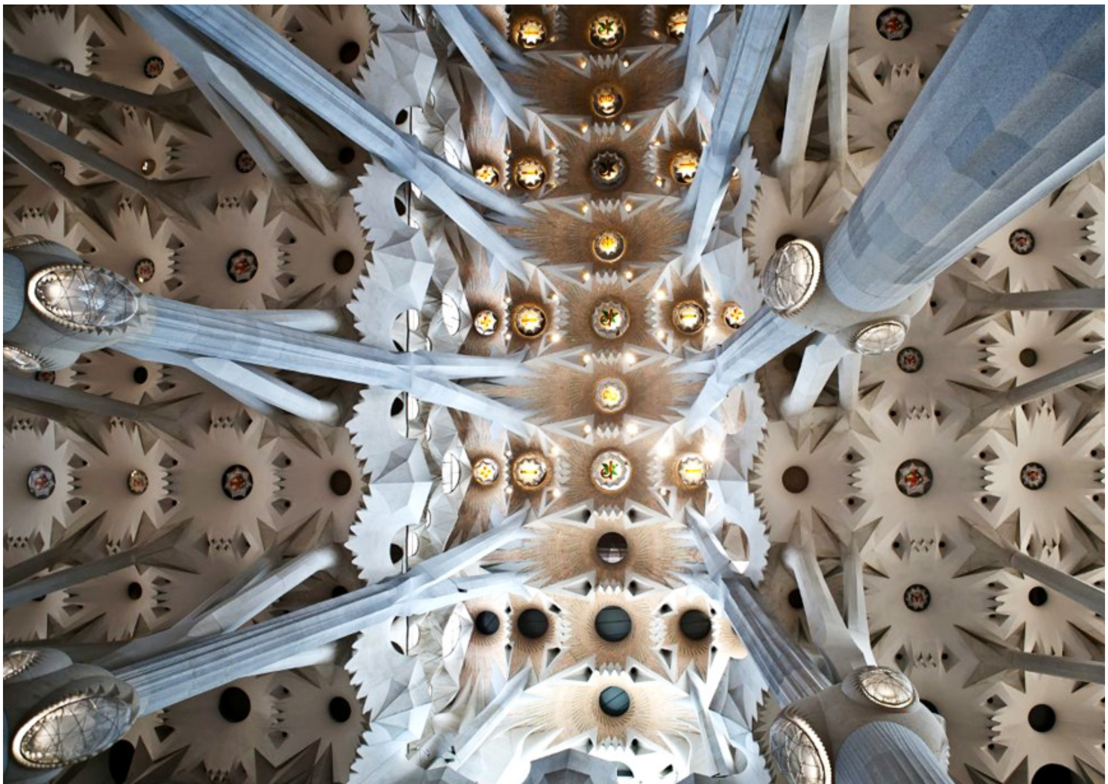

Fig. 104. A self-similar pattern of vault of the Sagrada Familia Church in Barcelona, one of most spectacular object of NeoGothic style and Art Nouveau. From: http://travel.nationalgeographic.com /travel /365-photos/sagrada-familia-barcelona-spain/