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Numerical applications of geometric-integral theory of underground mining effect of Professor Stanisław Knothe

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Abstract

The paper presents a general space-time variant of the geometric-integral theory of influences of Professor Stanisław Knothe. The theory describes the deformations of rock mass and surface area caused by underground mining. Then, calculation algorithms which allow to apply the theory in computer programs were presented. The first of the algorithms worked out in the 1970s by B. Drzęźla [10] assumes that the calculations are carried out in the polar coordinate system. It is characterized by extraordinary numerical effectiveness and it allows to approximate the mining process with any polygon. But the prediction results describe only the final deformation states, which is a kind of limitation. The second described algorithm, worked out by J. Białek [2, 3], uses the relations in a rectangular coordinate system. It allows to include in calculations the space-time evolution of the multi-panel and multi-bed underground mining exploitation. The programs also allow for the description of influence delay with respect to the completed mining exploitation, in line with the differential equation proposed by S. Knothe. The results of the prediction can involve the increment of deformations in any preset time interval, or extreme deformations in that time interval. Finally, the main reasons are presented, explaining why the application of S. Knothe theory has been popular in Poland and worldwide for 65 years.

Keywords: subsidence, strata mechanics, geometric-integral theory, numerical calculation methods of mining ground deformations

1. Geometric-integral theory of influences of Professor Stanisław Knothe – calculation of steady-state deformations – 65$^{\text{th}}$ anniversary of its formulation

One of the negative effects of underground mining involves the generation of continuous deformations of mining ground. Such deformations occur on large areas of the Upper Silesia Coal Basin, bringing about damage to numerous building structures. By forecasting such deformations, we can predict the negative impact of mining exploitation and make corrections in mining exploitation projects to reduce such negative impacts on building structures.

In Poland the predictions of continuous deformations of mining areas are carried out principally on the basis of the formulas of geometric integration theories of influences. The main idea of these theories involves the acceptance of the assumption that there exist so called function of influences $f(L,p_1,...,p_n)$, where L stands for a horizontal distance of the investigated point P on the mining area (in rock mass) from the element dS of the selected area S, and the acceptance of the superposition (summing up) principle of influences. The integration in these theories is carried out over the area being a projection of the mined-out panel on the horizontal plane.

$$w(P,.....) = -\iint_S a \cdot g \cdot f(L, p_1,..p_n) \cdot dS \tag{1.1}$$

where:

$w(P,\ldots)$ – subsidence of the ground at point P,

ag – maximum subsidence observed in a completed subsidence trough, when the dimensions of exploitation panel are appropriately big as compared to the exploitation depth,
$p_1,...,p_n$ – parameters of the theory of influences dependant on the properties of rock mass, exploitation depth, bed slope etc.

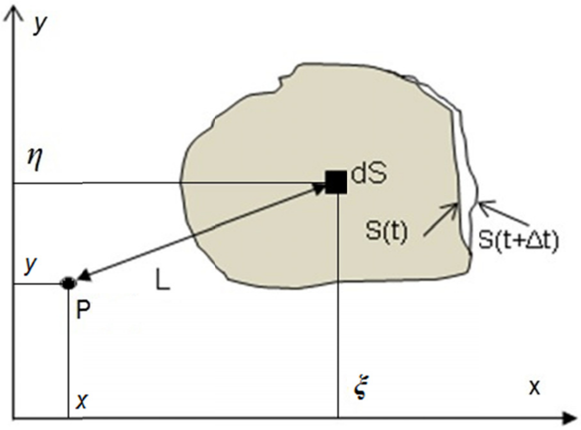


Fig. 1. Explanation of the symbols applied in equation 1

Exceptionally popular was the geometric integration theory of Professor Stanisław Knothe [13] published in the Archives of Mining and Steel Industry v.I, jour. 1, 1953, which after being complemented by Professor Witold Budryk [9] with the proposition to apply the hypothesis of S.G. Awierszyn for the description of horizontal movements [1], was also referred to as the theory of W. Budryk – S. Knothe. This theory has just celebrated its 65th birthday.

In the work [13] S. Knothe suggested the calculation of subsidence with the application of the influence function (1.2), which makes use of the Gauss function:

$$f(L,r)=\frac{1}{r^2}\exp\left(-\pi\frac{L^2}{r^2}\right) \tag{1.2}$$

where: r – parameter of S. Knothe theory referred to as radius of influence dispersion.

If we accept that the distance L from the calculated point $P(x,y)$ to the element of the surface dS of the coordinates ξ, η can be formulated by the relation $L^2 = (x-\xi)^2 + (y-\eta)^2$, and assuming that the deposition depth of seam h, thickness of the mined seam g and the subsidence coefficient a are variable, and the rock mass layers are sloped, which brings about the deviation of influences, then the general equation on the subsidence w of the point P described in the rectangular coordinate system has the following form:

$$w\big(P(x,y),S(t),..\big)=-\iint\limits_{S(t)}\frac{ag}{r(\xi,\eta)^2}\exp\left(-\pi\frac{\big[x-\big(\xi+p_x(\xi,\eta)\big)\big]^2+\big[y-\big(\eta+p_y(\xi,\eta)\big)\big]^2}{r(\xi,\eta)^2}\right)dS \tag{1.3}$$

where:
$S(t)$ – area of the mined-out seam whose dimensions and shape are generally the function of time t,
ξ, η – coordinates of the mining area element dS.

The length $r(\xi,\eta)$ of the influence dissipation parameter also referred to as the radius of main influence range is calculated from the relation:

$$r=r\big(\xi,\eta\big)=h\big(\xi,\eta\big)/\,\mathrm{tg}\beta \tag{1.4}$$

where:
$h(\xi,\eta)$ – depth of the mined seam or dS element of this seam,
$\mathrm{tg}\beta$ – the parameter of S. Knothe theory (tangent radius of main influence range β),
$p_x(\xi,\eta)$, $p_y(\xi,\eta)$ – components of the influence deviation vector p along the axes x, y.

The length of the deviation vector p is calculated from the relation (1.5):

$$p = h_k(\xi,\eta) \cdot \mathrm{tg}(k \cdot \alpha) \tag{1.5}$$

where:

- $h_k(\xi, \eta)$ – depth of the seam element dS in the sloped rock mass (Carboniferous),
- α – angle of dip of the seam,
- k – is a so called influence deviation coefficient which commonly assumes the value of $k = 0.7$.

Therefore, in order to apply the equation (1.3), we must know the geometry of exploitation (S,g), the location of the calculation point $P(x,y)$, value of the theory parameter tgβ and the value of subsidence parameter a and k.

Knowing the horizontal distribution of subsidence $w(x,y)$ defined by the equation (1.3), we can calculate slope components T of the subsidence trough profile:

$$T_x = \frac{\partial w}{\partial x}; \quad T_y = \frac{\partial w}{\partial y} \tag{1.6}$$

W. Budryk [9] proposed the calculation of horizontal components u_x, u_y of the dislocation vector basing on the Awierszyn hypothesis [1], which reads that the horizontal dislocation at point P is proportional to a certain constant B and to slope T. Hence we have:

$$u_x = -BT_x, \quad u_y = -BT_y \tag{1.7}$$

The equations (1.3) to (1.7) make up an unusually brief and at the same time general theory of rock mass movement which offers the description of the components of dislocation vector of rock mass movement effected by mining works.

For relatively small curvatures and slopes characteristic for subsidence troughs, the component values of vertical profile curvature of the subsidence trough calculated along the directions of axes x,y are calculated from simplified equations:

$$K_x = \frac{\partial^2 w}{\partial x^2}; \quad K_y = \frac{\partial^2 w}{\partial y^2}; \quad K_{xy} = \frac{\partial^2 w}{\partial x \partial y} \tag{1.8}$$

When we know the horizontal distributions of vectors u_x and u_y, based on Cauchy equations we calculate the horizontal components of strain tensor:

$$\begin{aligned}
\varepsilon_x &= \frac{\partial u_x}{\partial x} = \frac{\partial}{\partial x}(-BT_x) = -B\frac{\partial^2 w}{\partial x^2} = -BK_x; \\
\varepsilon_y &= \frac{\partial u_y}{\partial y} = \frac{\partial}{\partial y}(-BT_y) = -B\frac{\partial^2 w}{\partial y^2} = -BK_y; \\
\varepsilon_{xy} &= \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} = \frac{\partial}{\partial y}(-BT_x) + \frac{\partial}{\partial x}(-BT_y) = -2B\frac{\partial^2 w}{\partial y \partial x} = -2BK_{xy}
\end{aligned} \tag{1.9}$$

When we have horizontal components of strain tensor and the tensor of curvatures, we can calculate two horizontal components of main strains ε_1 ε_2 and the components of main curvatures K_1, K_2 of the subsidence trough profile.

It can be observed from the presented set of equations that in order to calculate the values of subsidence w, slope T and curvature K of the vertical profile of subsidence trough, we must know the values of parameters a, tgβ and k, and in order to determine horizontal dislocations U and horizontal strains ε, we must also know the values of parameter B.

For the equation (1.3) and for its derivatives, we must follow the principle of influence superposition resulting from the properties of definite integrals. If we consider the subsidence w effected by the exploitation of two longwalls $S1$ and $S2$, then, in line with the above principle we obtain the relation:

$$w(S_1 + S_2) = w(S_1) + w(S_2) \tag{1.10}$$

It means that the subsidence $w(S)$ calculated for the exploitation over the whole exploitation area $S = S1 + S2$ are equal to the sum of subsidence calculated separately for the plots $S1$ and $S2$.

The presented set of equations complemented with appropriate transformation equations is sufficient to produce numerical algorithms and computer programs calculating an arbitrary set of deformation indexes which characterize the influence of underground mining on rock mass and mining ground surface.

2. Numerical applications of S. Knothe theory

Apart from the simplest cases of exploitation which can be approximated with elementary shapes like half plane, rectangle, circle, ring, ring segment, the application of equation (1.3) or its derivatives necessitates the use of numerical methods and computer technology.

The works on computer software for the simulation of mining area deformation and the deformation or stresses taking place inside rock mass were initiated in 1970s by Bernard Drzęźla, who can be regarded as a pioneer in this area. At that time Bernard Drzęźla elaborated algorithms allowing for spatial character of deformation process and complex shape of abandoned workings, adapting to the domain of 'mining damage' a reach arsenal of means and notions from the field of theoretical mechanics and differential geometry. The author provided programs for most commonly applied in Poland geometric integration theories of W. Budryk – S. Knothe and T. Kochmański, as well as his own solutions. The achievements of B. Drzęźla have contributed to a qualitative change involving the prediction process of mining area deformations [10, 11, 17], and the calculation algorithms published by him have been used as a basis for the elaboration of numerous programs elaborated by other authors.

2.1. Basic calculation scheme proposed by B. Drzęźla – deformations of a steady state trough – integration in a polar coordinate system

In the handbook [17] B. Drzęźla published the description of computer programs elaborated by him for the deformation of mining ground together with a set of equations and algorithms applied in those programs. The most popular of those programs were made available as a reference source. The programs have been subjected to numerous transformations and developments made by other users.

B. Drzęźla assumed that the mined seam had the shape of horizontal seam of constant thickness. Then the author applied the equations of S. Knothe theory described in a polar coordinate system.

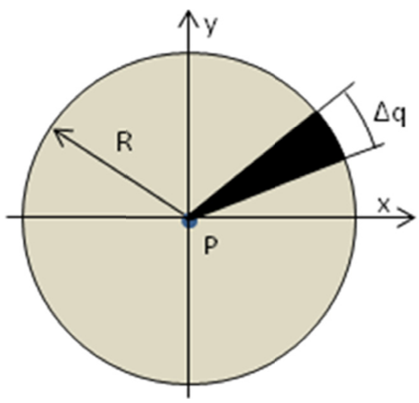


Fig. 2.1. Sector shaped exploitation

In the case of exploitation having the shape of a circular sector of the radius R and central angle Δq expressed in radians, the subsidence at point P is expressed with an unusually simple analytical equation (2.1):

$$w_k = -ag\left(1 - e^{-\pi\left(\frac{R}{r}\right)^2}\right)\frac{\Delta q}{2\pi} \tag{2.1}$$

The simplicity of this equation is one of basic advantages of S. Knothe theory.

B. Drzęźla derived (not so simple as the equation 2.1) equations allowing to calculate component slopes and curvatures for a circular sector. Using those equations, he proposed calculating the influence of triangle shaped exploitation of the apexes $P-(x_k,y_k)-(x_{k+1},y_{k+1})$ by approximating the area of that triangle with circular sectors presented in Fig. 2.2.

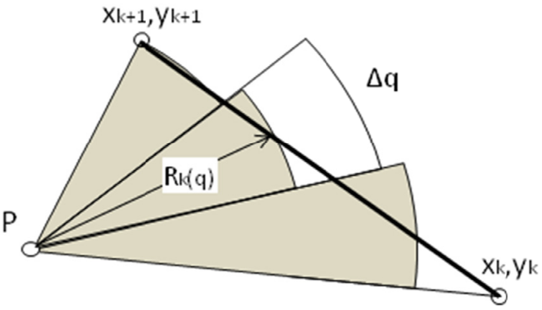


Fig. 2.2. Approximation of the triangle of the apexes $P-(x_k,y_k)-(x_{k+1},y_{k+1})$ with circular sectors

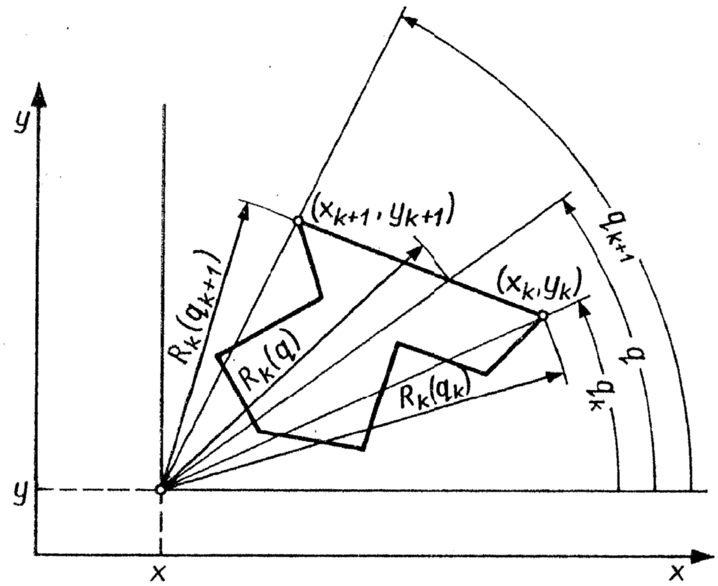


Fig. 2.3. Denotation for the calculation scheme assumed by B. Drzęźla [17]

Making use of this approximation for the description of polygon shaped exploitation of the apexes $1 \le k \le m$ (Fig. 2.3), the subsidence at point P is calculated from the equation:

$$w_k = -ag\sum_{k=1}^{m}\sum_{j=1}^{l}\left(1-\exp\left(-\pi\left(\frac{R_k(q_j)}{r}\right)^2\right)\right)\frac{\Delta q}{2\pi} \tag{2.2}$$

where:

$\Delta q = (q_{k+1}-q_k)/j$ – central angle of circular sector,

l – a number of circular sectors approximating the triangle's area,

$q_j = q_k + j\cdot\Delta q - \Delta q/2$ – angle between the axis x and the bisector of the j-th circular sector,

$Rk(q_j)$ – the length of the radius of circular sector changing along the side described by nodes $(x_k,y_k)\div(x_{k+1},y_{k+1})$.

The most important feature of the algorithm worked out by B. Drzęźla is its unusual numerical effectiveness. Owing to this effectiveness, at the time when the computer memory and computers speed were thousands times smaller, it was possible to enter data on completed mining of a whole coal mine (a few hundred longwalls) and to calculate the selected deformation indexes in the form of contour maps of the selected deformation indexes.

Another advantage of the calculation scheme presented in Fig. 2.3 is the possibility to allow for the exploitation of practically any shapes, which facilitates the numerical description of old completed mining which very frequently has complicated shapes.

Using the programs elaborated by B. Drzęźla, we can obtain deformation predictions for the final deformation states of mining ground after the completion of exploitation.

The most important programs in which B. Drzęźla implemented the theory of S. Knothe involve the programs allowing to obtain a tabular set of deformations for arbitrarily located calculation points (EDG3), programs which produce contour maps of the selected deformation indexes (EDG13) and programs which enabled a so called inverse analysis, i.e. the determination of theory parameters based on the geometry of completed mining and the results of geodesic surveys (EDG4).

Basing on the main algorithm of B. Drzęźla, in the years 1980-85, the author of the present paper elaborated the programs referred to as ENK3, ENK4 (Exploitation Sloped Final – the name had to have 4 characters) which in the calculations allowed for bed sloping and deviation of influences.

2.2. Author's programs – calculation of influences, allowing for the space-time development of mining exploitation

A very important practical issue in the field of deformation prediction of mining ground involves the description of influences with the time factor taken into consideration. The knowledge of this issue is necessary due to the following reasons:

- geodetic observations realized in successive measurement cycles provide the picture of the trough in different phases of its development, and therefore, in order to be able to compare the observations with the results of prediction calculations, we must be able to describe the subsidence at any stage of its development,
- the harmfulness of the influences on building structures is assessed basing on the calculated (less frequently measured) extreme in time horizontal deformations or/and curvatures of vertical profile, which necessitates the analysis of all development stages of subsidence troughs in time.

The problem of numerical description of mining development in time and resulting from that intermediate deformation states, basing on the differential equation of S. Knothe, was investigated in the works of J. Białek [2-4]. In the years 1978-80, the author elaborated computer programs of the symbols ED22, ED62, ED64, ED65, applicable on the computer Odra 1305, which in the prediction process allowed for both static and dynamic values of deformation. The author introduced the term of extreme deformations in the accepted time interval.

In effect of further works carried out after 1986, J. Białek elaborated a package of programs for the prediction of mining ground deformation with time factor taken into consideration, which was applicable on IBM-PC computers, and was known under the symbolic name of EDN-OPN.

The said programs [7, 8] have been systematically updated and developed both in terms of their functionality range, easier operation and the applied theoretical equations, calculation algorithms, applicability in successive versions of operation systems DOS-Windows and ability to work with AutoCAD [16]. Below we present some characteristic elements of algorithms resulting from the solutions of S. Knothe applied in these programs.

2.2.1. Quasi-static deformations (potentially possible) – integration in a rectangular coordinate system

When we ignore the influence of the insignificant delay involving the display of mining influence on the surface of mining ground, the development of influences in time can be treated as the effect of the increase of exploitation field in time $S(t)$ (Fig. 1.1). In contrast to the influences, which allow for the delay in time, the calculated in this way immediate (final) influences are further marked with an additional letter k.

In the general case (including also nonlinear theories of influences) the increment of subsidence in time Δt can be calculated as the difference of subsidence:

$$\Delta w_k(P(x,y),t,\Delta t..) = \iint\limits_{S(t+\Delta t)} -a \cdot g \cdot f(L,z,p_1,..p_n) \cdot dS - \iint\limits_{S(t)} -a \cdot g \cdot f(L,z,p_1,..p_n) \cdot dS \qquad (2.3)$$

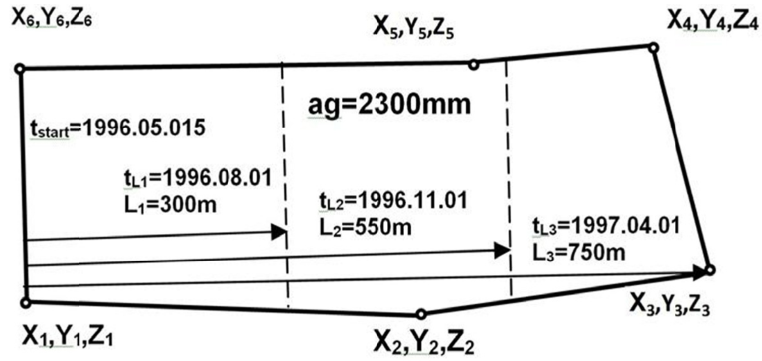
For the linear theory of S. Knothe, where the increment of subsidence Δw_k in the time interval Δt does not depend on the influences which occurred up to the moment t, it is sufficient to integrate over the field $\Delta S = S(t + \Delta t) - S(t)$:

$$\Delta w_k(P(x,y),t,\Delta t...) = \iint\limits_{S(t+\Delta t)-S(t)} -a \cdot g \cdot \frac{1}{r^2} \exp\left(-\pi \frac{L^2}{r^2}\right) \cdot dS \tag{2.4}$$

In the system of computer programs worked out by the author, the development of exploitation in time was allowed for, assuming that the panel (longwall) being exploited is treated as an arbitrary polygon defined by coordinates x,y (and possibly "z") of the polygon's apexes.

The development of exploitation in time is described in the following way (Fig. 2.4):

- it is assumed that the exploitation starts with a longwall cross-cut described with the side l÷m (the first and the last apex of the polygon), and also that the exploitation front is parallel to this side;
- we describe the advance of exploitation in time, giving the date of working startup T_{start} and one or more pairs of numbers defining the longwall's advance L_j and the date when the advancement was reached T_j is.



T_{start} – date of longwall startup
L_j,T_{Lj} – pairs of numbers defining the longwall's face advancement length L_j and the date T_j when the length will be reached

Fig. 2.4. Description principles of the in-time changing geometry of the single longwall

In effect of the applied description of exploitation in time, it is possible to obtain the predictions of deformation indexes calculated for arbitrarily defined time intervals.

Assuming a coordinate system x',y' whose centre is located at the calculation point $P(x,y)$, rotated in the way ensuring that the axis x' is parallel to the side 1÷m describing the longwall cross-cut, and dividing the plot into narrow strips of the width proportional to the longwall advance obtained in the time interval Δt, we are approximating the plot polygon into narrow rectangles which bring about the rise of deformation in this time interval.

When we assume that the bed is horizontal, the increase of subsidence of the point P located in the centre of the coordinates x',y' can be expressed as follows:

$$\Delta w_k\left(P(0,0),t_i,\Delta t\right) = -ag \int\limits_{\frac{X'_L}{r}}^{\frac{X'_P}{r}} e^{-\pi x^2} dx \cdot \int\limits_{\frac{Y'_D}{r}}^{\frac{Y'_G}{r}} e^{-\pi y^2} dy \tag{2.5}$$

where:

X_L, X_p – coordinates x of the left and right side of the calculation rectangle,
Y_D, Y_G – coordinates y of the bottom and top side of the calculation rectangle,
i – number of time interval in the accepted time system t.

We can see that the calculation of the increment of subsidence effected by the exploitation of the rectangular plot is narrowed down to the calculation of the product of two single integrals, which is several dozen times faster than the calculation of a respective surface integral.

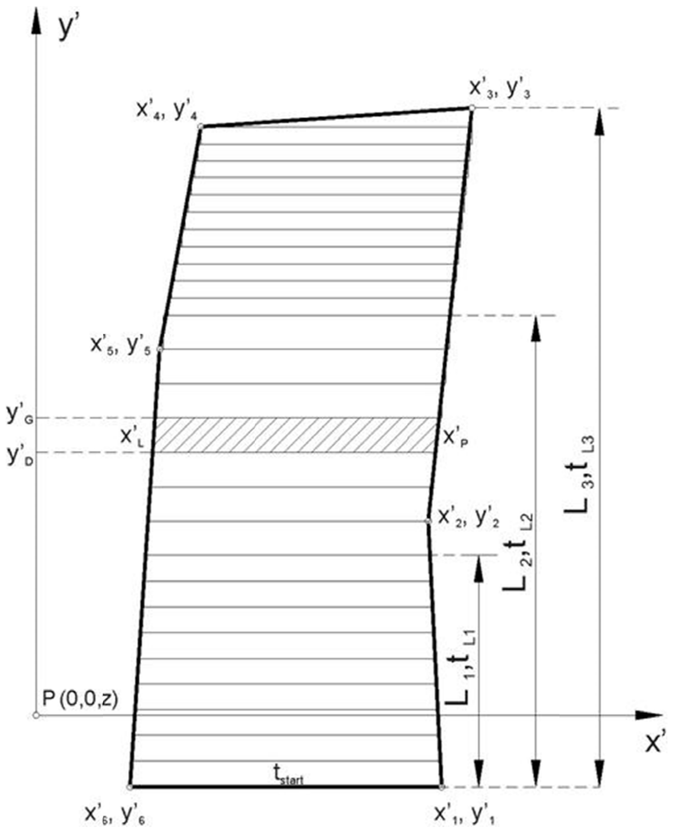


Fig. 2.5. Longwall plot after the transformation into the system x',y' divided into narrow rectangles of the height $\Delta y' = Y_G - Y_D = v \cdot \Delta t$ proportional to the speed of longwall front advance v

Because of numerical applications, it was one of the most important advantages of S. Knothe theory. It was the application of Gauss function which, in the case of rectangle shape type exploitation, enabled to transform the equation (1.3) (double integral) into the product of single integrals. I personally do not know any other function which would be capable of doing it and which would at the same time have the properties to be applied as the function of influences.

We can add here that equally easy equations can be used to calculate the increase of the remaining deformation indexes $\Delta T'_{xk}$, $\Delta T'_{yk}$, $\Delta K'_{xk}$, $\Delta K'_{yk}$, $\Delta U'_{xk}$, $\Delta U'_{yk}$, $\Delta\varepsilon'_{xk}$, $\Delta\varepsilon'_{yk}$, $\Delta\varepsilon'_{xyk}$ which must be then transformed to the original, common for all panels system x, y, additionally allowing for the necessity to calculate them in the directions rotated by the value of calculation direction preset in the task.

The summed up and appropriately memorized calculated increment values of deformation in successive time intervals Δt allow to follow the changes of deformation in time, and also to detect the values of deformations extreme in time. These are potentially possible deformations (quasi-static), calculated without time delay taken into consideration.

2.2.2. Description of non-steady-state influences with the use of S. Knothe differential equation

Of primary importance for the description of non-steady-state subsidence troughs, where we must allow for the development of exploitation in time and the delay of influences, is the differential equation (2.6) proposed by S. Knothe [14]. According to this equation, the speed of subsidences dw/dt at time t is proportional to the value of parameter c and to the difference between the subsidence $w(t,\ldots)$ taking place at time t and the subsidence which can be defined as potentially possible subsidence at time t, i.e. the subsidence which would occur if the subsidence process was running without delay in time.

$$\frac{dw(t,..)}{dt} = c\left[w_k(t,..) - w(t,..)\right] \tag{2.6}$$

where: c[1/time] – index of subsidence speed, also called time index.

One of the solutions of differential equation (2.6) is the formula (2.7):

$$w(t,..) = \int_0^t \frac{dw_k(\tau,..)}{d\tau}\left[1 - \exp\left(-c(t-\tau)\right)\right] d\tau \tag{2.7}$$

Time τ is the exploitation time of surface element dS, which would potentially (if the subsidence was without delay in time) bring about the increment of subsidence $dw_k(\tau)$. The discretization of this equation consists in replacing the differentials $dw_k(\tau,..)$ with a sequence of finite increments Δw_{ki} calculated for time intervals from $t = \Delta t$ to $t = n \cdot \Delta t$. The calculation method of these increments was described in the previous chapter. Hence, we obtain a series (2.8):

$$w(t = n \cdot \Delta t,..) = \sum_{i=1}^{n} \Delta w_{ki} - \sum_{i=1}^{n} \Delta w_{ki} \cdot e^{-c \cdot \Delta t (n-i)} \tag{2.8}$$

In order to find deformations extreme in time (it does not involve subsidence which change in a monotonic way), we must know the values of deformations for successive times $t_i = i \cdot \Delta t$, where $1 \le i \le n$, so we must repeat the calculations n times with the equation (2.8). A fast calculation method of successive expressions of the series 2.8) is described in works [2],[4].

2.2.3. Functionality range of programs for deformation calculations

The potential of the author's package of computer programs for the realization of prediction calculations with the application of the presented algorithms is most effectively illustrated by the presented window of control program in the figure 2.6, having the symbolic name EDNOPN.

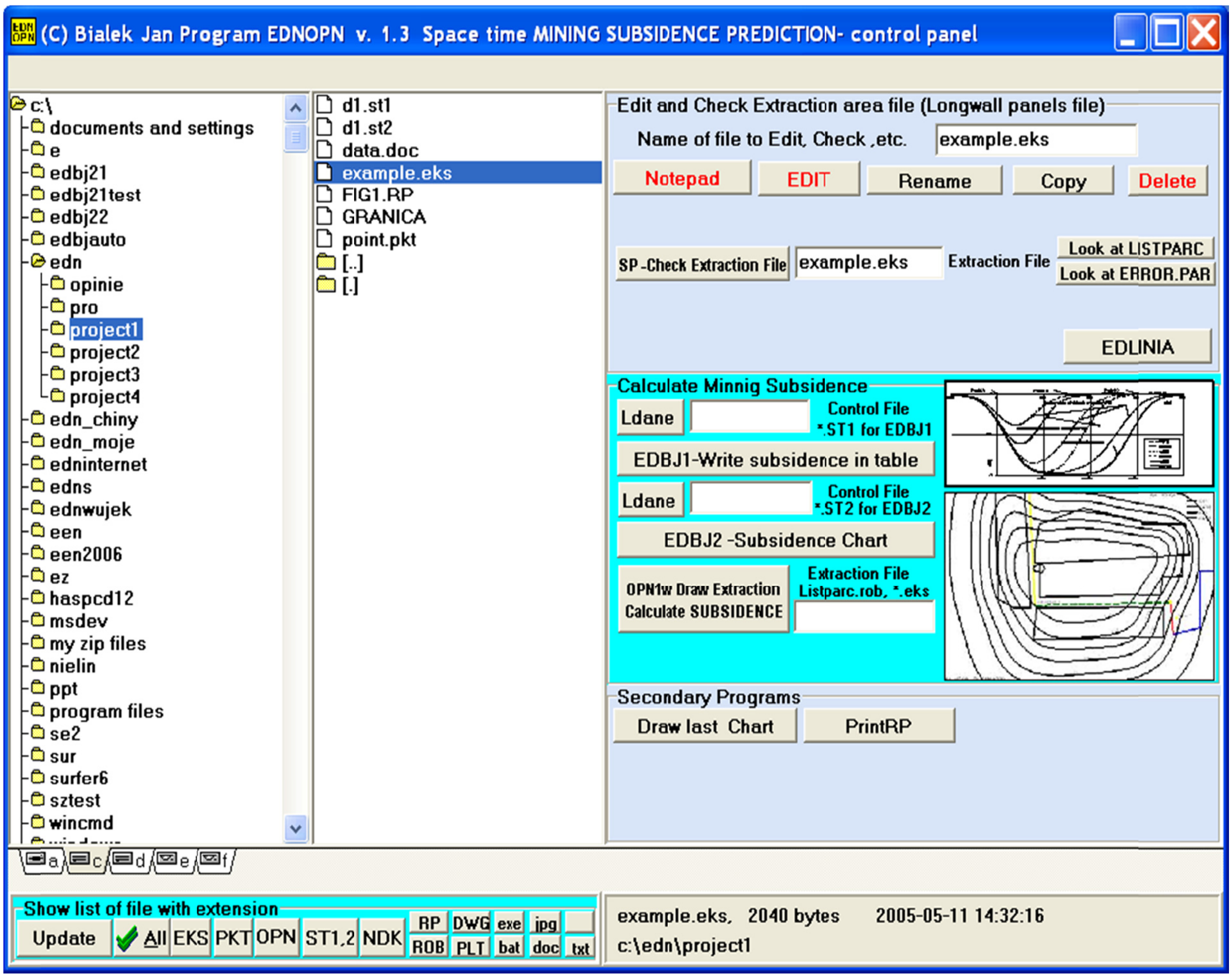


Fig. 2.6. Window of the EDNOPN control application that serves as computational environment for applications that perform the actual prediction calculations

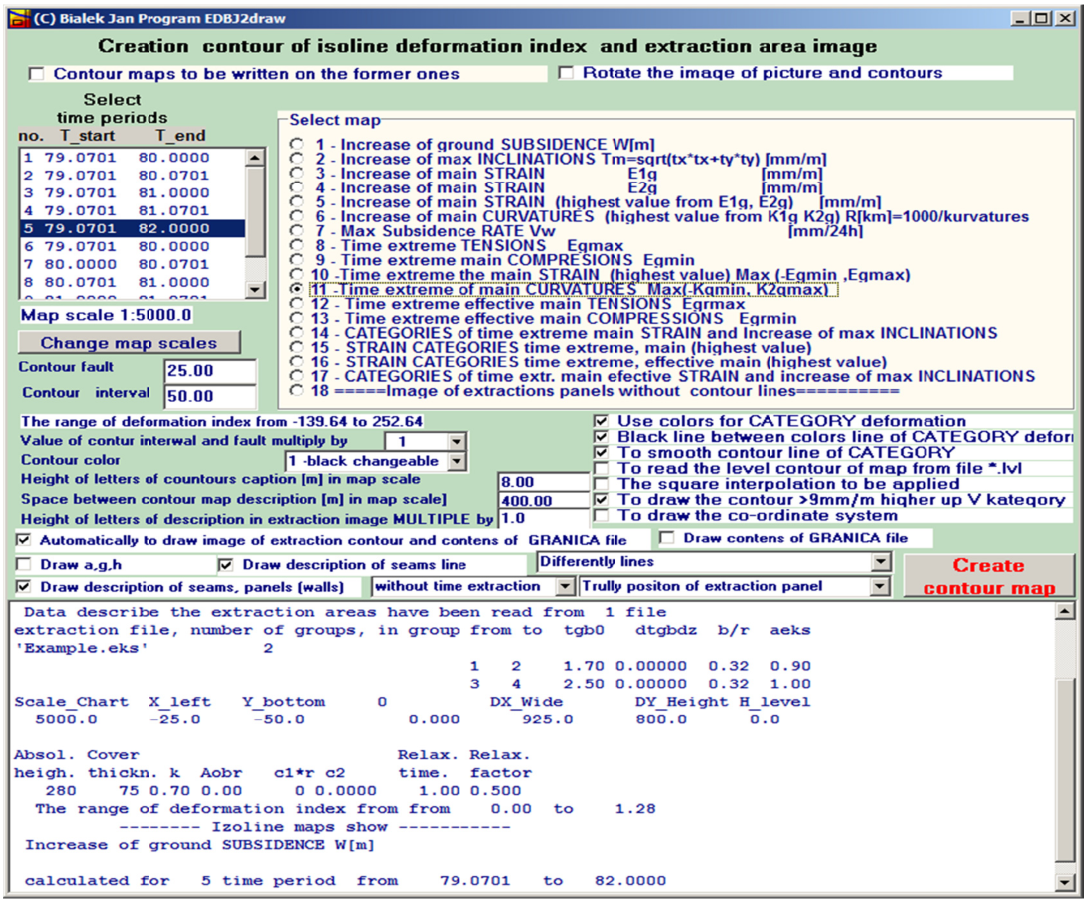


Fig. 2.7. Window of EDBJ2draw application – options selection of the designed drawing of contour lines of deformations along with the drawing's background

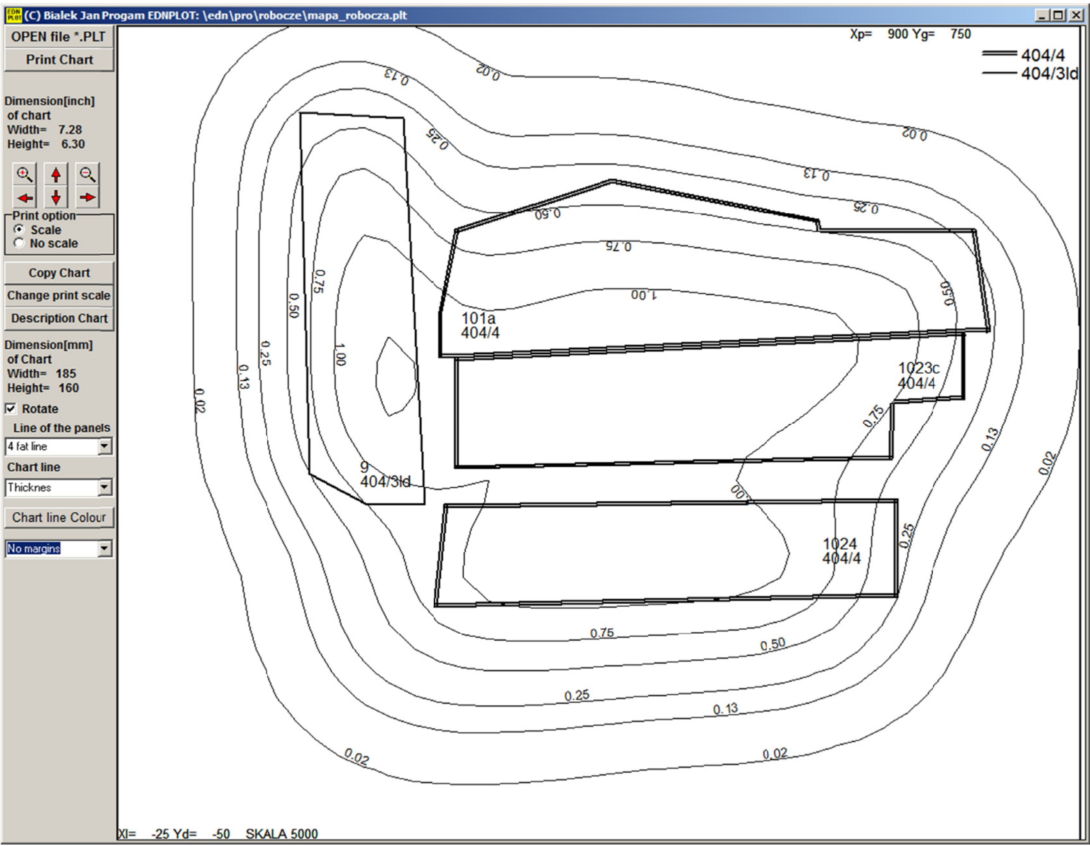


Fig. 2.8. Window of EDNPLOT application invoked automatically from EDBJ2draw application

The window consists of 3 panels. The left panel is the catalogue panel. In the middle panel, we can see the files being the content of this catalogue. The keys on the right panel invoke the successive programs of the package. Three thematic groups have been singled out in this panel:

1. A group of programs for checking and changing the data about mining exploitation.
2. A group of programs realizing prediction calculations.
3. A group of auxiliary programs, including the programs realizing a so called inverse analysis, i.e. programs determining the parameters of the theory of influences based on known results of geodesic surveys.

The most important programs calculating the deformations involve such programs as EDBJ1 – calculation of deformations in a tabular form, a series of programs EDBJ2 (Fig, 2.7 and Fig. 2.8) which generate final contour maps of the selected deformation indexes together with contour lines of mining exploitation and with the pictures of essential elements of mining ground, and the program OPN1w generating the survey-geological opinion in the format required by mining offices.

3. Advantages of S. Knothe theory

The basic advantage of this theory is manifested by its unusual simplicity characteristic for all geometric integration theories. In the case of S. Knothe theory, there is also a very accurate selection of the influence function in the form of Gauss function. It was only the Gauss function that could be transformed from the function of two variables into the product of two homogeneous functions of one variable (3.1).

$$e^{-\pi L^2} = e^{-\pi\left(X^2+Y^2\right)} = e^{-\pi X^2} \cdot e^{-\pi Y^2} \tag{3.1}$$

Owing to the dependence (3.1), the spatial solution (1.3) can be narrowed down to the product of flat solutions (2.5).

As unusually accurate can be viewed the parametrization method of the influence function proposed by S. Knothe (1.2). The parameter r from this function, referred to as radius if influence dissipation, is at the same time the radius of main influence range, where according to the theory, the subsidence are smaller than 0.6% of the maximum subsidence.

The length of the radius r is connected with a very important dependence which allows us to determine its length basing on the measured maximum slope $T_{\max}$ and the subsidence $w_{\max}$ of the complete subsidence trough:

$$T_{\max} = \frac{w_{\max}}{r} \quad \rightarrow \quad r = \frac{w_{\max}}{T_{\max}} \tag{3.2}$$

In case of Upper Silesian Coal Basin (USCB) we must add that the most commonly accepted length of the radius r is equal to the half of the depth of the analyzed seam. The simplicity and obviousness of these dependences contributed to the popularity of S. Knothe theory.

The peak of numerical effectiveness is expressed by the equation (2.1) describing the subsidence effected by a circular sector-shaped exploitation. It is a closed analytical formula consisting of one elementary function – it's just that simple. The application of this equation leads to a very fast algorithm in numerical realization (2.2), owing to which we can calculate the subsidence effected by the exploitation of bed of any shape.

Another page in the development history of deformation description in time involves the differential equation (2.6) proposed by S. Knothe and his differential expressed by the equation (2.7). I think there is no equally simple and at the same time so general and universal description of a phenomenon. We must add here that the equation (2.7) has been subjected to many modifications. For the calculation of $dw_k(\tau)$, we can apply different influence theories, not necessarily the equations of S. Knothe, and in the form of time function $F(t-\tau) = 1 - \exp(-c(t-\tau))$ there might be for example its linear combinations [12].

Finally, we must add that the equation (1.3) can be treated as one of possible solutions of the stochastic medium theory of J. Litwiniszyn [15], which means that this equation has deeper theoretical reasoning, and furthermore, it describes well the subsidence of loose medium.

4. Summary

During the 65 years of existence the theory of Professor Stanisław Knothe has proved its usefulness as a tool facilitating the assessment of the influences of completed and planned mining operations. It has inspired various research studies and scientific publications in the field of mining area deformation, and a great number of researchers contributed to its development and improvement.

For me the theory of S. Knothe is a set of unusually effective equations which I have applied in the computer programs worked out by me. Initially I created the programs as a set of tools which would enable the realization of complex prediction problems, and in time they assumed the form of commonly available applications.

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