Modeling of land subsidence caused by salt cavern convergence applying Knothe's theory

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Abstract

It has been years since Knothe's theory was used for modeling deformations and strains generated in the rock mass and on the surface for various types of raw minerals, salt including. The qualitative congruence of the Knothe's model with the observed form of subsidence in the salt mine areas can be proved in all places where such observations were made. In Poland, subsidence troughs accompanying salt domes are monitored in Wieliczka, Bochnia, Kłodawa and Sieroszowice. Based on many years' surveys in the Wieliczka mine proper mathematical model, calculation algorithm and parametrization method could be selected. The usability of theoretical formulas originally written for coal beds was verified on the example of a historical salt mine. The present functionality of modified Knothe's model was shown for salt rock on the example of predictions of vertical deformations and strains in the "Wieliczka" Salt Mine.

Keywords: Modeling of land subsidence, salt mining, convergence, Knothe's theory

1. Introduction

1.1. From Knothe-Budryk's theory to the implementation of calculation model for salt domes

The Knothe-Budryk's theory was adapted to predicting mining-induced impact and for over 60 years the theoretical solutions could be tested in practice [8]. In this period the geological and mining conditions kept on changing, therefore modifications were obviously needed. The implementation of this model to the salt dome conditions was initially almost unthinkable. The theoretical model was worked out for powdery or brittle rocks, not for rheological media. In this sense the presented modification of the primary model did not meet original assumptions of the continuity of medium. The first works on the adaptation of the model to salt dome environment date back to the early 1980s. Sroka and Schober proposed a two-parameter function of time, thanks to which the rate of convergence of salt caverns could be taken into account [12]. Authors made use of this function (4) in their paper. In Germany this solution was used for salt domes by Schober and Sroka and then tested in the Asse mine by Hejmanowski. The results were modified for mining elements and implemented since eighties in the Polish conditions [6]. A few applications of the Knothe's model were used in reference to the salt dome in the historical, 700 years old "Wieliczka" Salt Mine.

1.2. Geology of Wieliczka salt dome

The whole recognized Wieliczka deposit extends eastward along the edge of the Carpathians. It is about 10 km long, on average 1 km wide, and 15-350 m deep. Finally, the Wieliczka deposit splits into the upper part (lump), levels I-III and lowermost (deposit), levels III-IX [13].

The ultimate form of the deposit is a result of overthrust tectonics of the Carpathians. In the course of these processes the upper part was deformed in the northern direction and the lower part was folded. Roughly speaking, about 9 379 957 m³ was extracted from "Wieliczka" Salt Mine in its 700-years' history. Wieliczka rock mass was perforated with numerous unbackfilled workings, which remained separated with virgin sections of different size. This rock was plastic therefore post-extraction spaces underwent long-lasting, advancing contraction processes and a dense network of interfering local subsidence troughs was generated. Consequently, a subsidence trough, considerably regular in time and space, was formed on the surface.

Wieliczka rock mass shows specific elasto-plastic (rheological) properties causing that the postextraction voids naturally undergo slow contraction under the load of the onlying beds (rock mass pressure). In the initial period, only continuous plastic strains are generated. After the yield point of rocks surrounding the working is exceeded, discontinuous deformations (cracked rock mass) are the case, the rock masses fall off onto the workings and the caving processes (rock slide) intensify in the closest vicinity of the working. The caving continues till the moment the roof is self-supported (self-backfilled) and rests on the loose rock material which fell down from the roof and the side walls.

1.3. Measurement of surface and rock mass in "Wieliczka" Salt Mine

The "Wieliczka" Salt Mine ceased its mining activity in 1996. Since that time the process of applying prevention measures (backfilling and rebuilding) continues to maintain the mine and keep it open to the tourists. This requires constant monitoring of deformations and strains, being a consequence of the advancing convergence of chambers and galleries. Since 1926, geodesic surveys of strains and deformations have been conducted in the mine and on the surface. The network of points where surface deformations are measured totals to 1165 points, out of which 421 are scattered and 744 points make the geodesic lines on the surface [13]. The monitoring also encompasses the measurement of vertical and horizontal convergence in underground workings, unfortunately mainly within the lump deposit. The bases on which the convergence measurements were conducted were successively developed; presently about 513 bases exist in 73 workings [13]. In this group are mainly chambers of the Tourist Route, Krakowskie Żupy Museum, Sanatorium and shallow chambers. The monitoring is extended on the rock mass, besides the underground leveling network was measured based on the height transposition through the available mine shafts at all accessible levels.

2. Model and its parametrization

The calculation model used for modeling strains in salt mining evolved with time from the basic solution, i.e. Knothe's theory with Awierszyn's hypothesis, and simple, one-parameter function of time (Knothe) to the solution adjusted to the salt dome conditions [8]. It is based on the generalized Knothe's model with modifications and improvements. The solutions used by Authors rely on the division of post-mining void into elements, which are ascribed geometrical data and a set of parameters of the model [6] (Fig. 1). Each



Fig. 1. Schematic of deposit division into extraction elements [10]

extraction element 'i' generates an elementary subsidence trough on the surface, and the vertical deformation in such a trough can be determined for an arbitrary point on the surface 'j' with equation (1).

$$\Delta S_{j,i}\left(d_{i,j},t\right) = V_i\left(t\right) \cdot \phi\left(d_{i,j},H\right) \tag{1}$$

where:

- $\Delta S_{j,i}$ vertical displacement in elementary subsidence trough caused by volume extraction in i-th element of deposit,
 - i number of a subsequent individual extraction element,
 - j number of a single calculation point,
- $V_i(t)$ volume of post-extraction void in the *i*-th extraction element,
 - H mining depth,
 - t time difference between mining date and date of prediction,
- $d_{i,j}$ (horizontal) distance between the calculation point and the centre of the extraction element,

 $\phi(d_{i,j}, H_i)$ – influence function, given with equation (2):

$$\phi\left(d_{i,j}, H_i\right) = \frac{1}{r^2} \exp\left(-\pi \cdot \frac{d_{i,j^2}}{r^2}\right)$$
(2)

where: r – parameter of dispersion of mining impact, according to Knothe's model.

The deformation indices are obtained by summing up the impacts of particular extraction elements and accounting for the assumed theoretical model. Vertical deformations evoked by extraction in the whole deposit (chambers or many chambers) can be determined with equation [10] (3):

$$S_j(t) = \sum_{i=1}^{i=N} \Delta S_{j,i}(d_{i,j}, t)$$
(3)

where:

 $S_i(t)$ – total, momentary vertical deformation of point *j*,

N – number of mining elements.

Thus the use of the generalized Knothe's model for calculating strains in salt dome is connected with the geometry of extraction elements and proper parametrization of the model. Importantly, the volume of the elementary trough, corresponding to the volume of the extraction element, depends on time and two time parameters. Function of time λ , which was used in the model is a two-parameter Sroka-Schober function [12, 9] (4):

$$\lambda = 1 + \frac{\xi}{\vartheta + \xi} e^{-\vartheta \cdot t} - \frac{\vartheta}{\vartheta - \xi} e^{-\xi \cdot t}$$
(4)

After using it to the model of extraction elements we get dependence (5), which describes momentary volume of post-extraction void in the extraction element [6]:

$$V_i(t) = a_i \cdot g_i \cdot E_i \cdot \left[1 + \frac{\xi}{\vartheta + \xi} e^{-\vartheta \cdot t} - \frac{\vartheta}{\vartheta - \xi} e^{-\xi \cdot t} \right]$$
(5)

where:

- a_i local extraction coefficient,
- g_i local height of extraction element,
- E_i area of void in a given extraction element (Fig. 2),
- ξ relative, volumetric rate of convergence [1/year],
- 9 relative delay in caprock [1/year].

Although the mining chambers of the 'Wieliczka' Salt Mine had an irregular shape, far from regular cuboids, a model based on extraction elements was used for describing complex geometry of the chambers. In this calculation model the height of the extraction elements could be regulated depending on the geometrical

complexity of the mining chamber. The vertical localization of all extraction elements and their height could be determined through interpolation. In this way the theoretical description was adjusted to the actual shape of the extraction. The Higher Drozdowice chamber (1841) was outlined in Fig. 2. It was viewed from the second lower level and the shape was drawn on the basis of a mining map. Then its geometry was divided into extraction elements (their sides were 5 m long).



Fig. 2. Division of salt chamber Drozdowice Wyższe into extraction elements

Each extraction element was ascribed a volume of part of the chamber which was encapsulated by that element. This volume was determined as a cuboid, the base of which corresponds to the common part of the chamber and extraction element, whereas the height was defined on the basis of the working's geometry (extracted height), way in which the chamber was filled (filling of void or way in which the chamber was protected), parameters of volume convergence in a given place of the deposit and time which elapsed from the last stock taking. All geometrical data of the element are ascribed to the middle of the element localized at the chamber's floor level. In this way the post-mining voids can be quite accurately (as for deformation modeling) geometrized.

As mentioned, correctly performed calculations depend on the parametrization of the selected calculation model so that the geometrical model of a complex rock mass and the deformation in time can be accurately described. The classical Knothe's model made use of two parameters "a" and "r". After numerous modifications and improvements aimed at adjusting this model to various mining conditions, various types of deposits and processes of strains, the following parameters were used:

- $tg\beta$ tangent of range of major impact in Knothe's model (knowing the anisotropy of the rock mass one can introduce various values of this parameter in various directions),
- a parameter depending on the mining system, way in which voids are protected and type of material used (backfilling material); value of this parameter is determined individually for each chamber taking into account its planned destination; for a chamber which was not backfilled a = 1.0,
- *B* proportionality ratio of vertical and horizontal strains, resulting from Awierszyn's hypothesis (the coefficient depends on the depth of the chamber),

- μ parameter describing deviation of subsidence trough in a function of build of the rock mass; one can hardly speak about the tilt of a salt dome, yet with this parameter we can take into account the deviation in, e.g. the results of geodesic monitoring,
- ξ parameter of a two-parameter function of time which describes the convergence of a given chamber (extraction element), depending on numerous mining-geological factors [12],
- ϑ parameter of function of time describing the transition of strain from a converging chamber to the assumed calculation horizon,
- n parameter determining the variability of the major impact radius in the rock mass with the change of distance from the chamber to the calculation horizon (its value is significant only for the calculation of deformations in the rock mass) [11],
- r_s radius of major impact on the level of roof of the extraction chamber; it allows for taking into account deformations in the chamber area (important for, e.g. modeling of deformations and strains in the rock mass) [11].

Models with so many parameters can be unstable, therefore parametrization based on the results of measurements of various elements of deformation field is so important. Advantageously for the elementsbased strategy, various convergence rates of chambers (ξ) in various parts of the mine can be taken into consideration. The measurements of deformations and strains in the rock mass and on the surface are very important when determining the range of impact (tg β), radius of scattering in the chamber roof (r_s) and deviation coefficient (μ). In this situation various parameters can be determined and introduced as constants in given geological-mining conditions. This procedure and reverse analysis method bring about a model with optimum values of applied parameters [7].

3. Exemplary modeling

3.1. Modeling of rock mass strains in the Chapel of St. Kinga

The Chapel of St. Kinga is one of the places where modified Knothe's model was applied for modeling displacements and strains in the Wieliczka rock mass [1]. The calculation model was parametrized based on the results of many years' surveys. The displacements and strains were modeled for a group of chambers localized below and near the Chapel (Fig. 3).



Fig. 3. Spatial system of chambers accounted for displacements and strains predictions for the Chapel of St. Kinga [1]

The calculations were performed with the use of own computer program. The calculations of historical data brought about an exemplary distribution of horizontal strains in the directions of assumed axes of local coordinates system (Fig. 4, 5).



Fig. 4. Horizontal strains along OX axis [1]



Fig. 5. Horizontal strains along OY axis [1]

The distribution of modeled horizontal strains complied with observed strains and the consequences they generated in the Chapel. It also correlated with the results of numerical modeling. Above results prove the experimental assumptions [1].

3.2. Modeling of strains and deformations in the Regis Shaft

Modeling of salt rock mass strains in the "Wieliczka" Salt Mine was also realized for one of the oldest mine shafts. It was built and maintained for centuries, since the middle of the 14th century. The Regis Shaft is an upcast shaft. At present the shaft is 196.67 m deep, reaching level V. The modeling focused on historical strains and their condition as of 2013 to approximately point areas requiring special care, monitoring or rebuilding.

The results of calculations of horizontal and vertical strains along the Regis Shaft are presented in Fig. 6 [3].



Fig. 6. Plots of horizontal and vertical strains along the Regis Shaft [3]

By the year 2013 the historical vertical strains could possibly reach the value of ca. $\varepsilon_{z \max}^{(-)} = -30$. These were compressive strains. Their high values at the height of +140 m a.s.l. was connected with the subsidence of the upper mine levels as compared to the lowermost, where the subsidence was attenuated. This effect is also evidenced by considerably high horizontal strains and tensile strains in this zone. The age of the shaft justifies such high values.

4. Concluding remarks

The applied calculation model, based on formulae derived by professor Knothe and professor Budryk in the 1950s, is technically the only fully effective method of predicting strains and deformations in salt domes. Salt rock structures are spatially extensive, geologically and geomechanically very complex, and that is why they hinder of make numerical modeling almost impossible, especially when the development of strains in time is involved. The Knothe's geometrical-integral model is relatively simple and so very efficient, therefore the solution presented on the example of the "Wieliczka" Salt Mine could be successfully implemented for other salt deposits in Poland, i.e. in "Kłodawa" Salt Mine (prediction for Deposit Development Plan [2]), "Bądzów" deposit in KGHM (prediction for Deposit Development Plan [4]), "Bochnia" Salt Mine (prediction for 25 years [5]). It should be emphasized that these projects were also realized for other types of deposits, and no matter whether the accumulation assumed the form of a dome or bed or seam, the modeling was successful.

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