

The Application and Development of Knothe Influence Function in China

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Abstract

The Knothe influence function method was introduced to China in 1965 and began to abundant research and widely applied from 1980s. Based on the Knothe influence function method, the prediction model of surface movement and deformation of flat seam mining is established, then the calculation programs and drawing software are developed. The relationship between surface movement calculation parameters and geological mining conditions is systematically summarized, and national technical regulations and standards are compiled. The method has been widely applied in special coal mining and got enormous economic benefits, meanwhile, the method is applied in metal mine, oil and gas, groundwater, halite mining and underground engineering. In recent years, the Chinese scholars enrich and develop the probability integral method, and the prediction model of surface movement and deformation with special mining conditions, such as extremely inadequate mining and steeply inclined seam mining is established. Through introducing the Knothe influence function method of application in China, the 100th birthday of Professor Knothe and the great contribution to the mining subsidence prediction is commemorated.

Keywords: Knothe influence function; surface movement deformation; subsidence prediction model

1. Introduction

Prediction of surface movement and deformation is necessary for mining under the surface structures, many scholars from different countries have done lots of researches and some specific prediction methods are found [1-3]. Based on Chinese references Knothe influence function is introduced in Chinese mining subsidence prediction. The Knothe influence function is called probability integral method, because it has probability integral in the equation. Chinese scholars believe that the stochastic medium theory is the basic of probability integral method. A book named “Basic Law of Surface Movement in coal” by Chinese scholars Liu Baochen and Liao Guohua introduced Knothe influence function in 1965 [4], it established the probability integral method based on the stochastic medium theory. It has been became one of the most extensive prediction methods for mining subsidence prediction.

2. Calculation model of probability integral method

Surface movement and deformation prediction model of flat seam mining based on Knothe influence function is established and the prediction system of probability integral method is formed.

2.1. Prediction model of surface subsidence at any point

Mining length of flat rectangle seam in trend is l , the width in tendency is L , the mining area are $x \in [0, l]$, $y \in [0, L]$, respectively. The working face and calculation coordinate system is following as Fig. 1. Considering probability direction of x, y is independence, so the subsidence value of the point $A(x, y)$ on the surface is following:

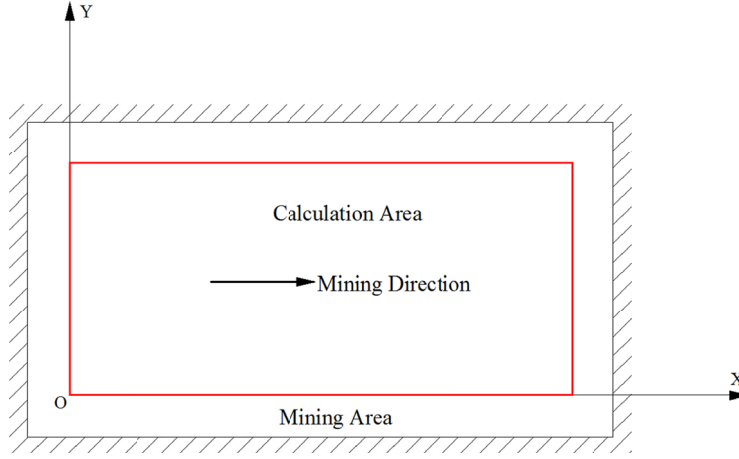


Fig. 1. The working face and calculation coordinate system

$$W(x, y)_A = W_{\max} \iint_F f(x, y) dF = W_{\max} \iint_F f(x) f(y) dx dy \quad (1)$$

Knothe influence function is applied:

$$f(x) = \frac{1}{r} e^{-\frac{\pi x^2}{r^2}}, \quad f(y) = \frac{1}{r} e^{-\frac{\pi y^2}{r^2}} \quad (2)$$

The lower left of the working face is as coordinate origin, then the subsidence value of $A(x, y)$ is following:

$$W(x, y)_A = \frac{W_{\max}}{r^2} \int_{-x}^{l-x} e^{-\pi \left(\frac{x}{r}\right)^2} dx \int_{-y}^{l-y} e^{-\pi \left(\frac{y}{r}\right)^2} dy \quad (3)$$

$$\begin{aligned} W(x, y)_A &= W_{\max} \times \frac{1}{2} \left\{ \left[\operatorname{erf} \left(\sqrt{\pi} \frac{x}{r} \right) + 1 \right] - \left[\operatorname{erf} \left(\sqrt{\pi} \frac{x-l}{r} \right) + 1 \right] \right\} \\ &\quad \times \frac{1}{2} \left\{ \left[\operatorname{erf} \left(\sqrt{\pi} \frac{y}{r} \right) + 1 \right] - \left[\operatorname{erf} \left(\sqrt{\pi} \frac{y-L}{r} \right) + 1 \right] \right\} \\ &= \frac{1}{W_{\max}} [W(x) - W(x-l)] [W(y) - W(y-L)] \end{aligned} \quad (4)$$

Simplifying the equation:

$$W(x, y)_A = \frac{1}{W_{\max}} W_{(x)}^0 W_{(y)}^0 = W_{\max} \cdot \frac{W_{(x)}^0}{W_{\max}} \cdot \frac{W_{(y)}^0}{W_{\max}} = W_{\max} C_{(x)} C_{(y)}$$

In the equation: $C_{(x)} = \frac{W_{(x)}^0}{W_{\max}}$, $C_{(y)} = \frac{W_{(y)}^0}{W_{\max}}$

$$C_{(x)} = \frac{1}{2} \left\{ \left[\operatorname{erf} \left(\sqrt{\pi} \frac{x}{r} \right) + 1 \right] - \left[\operatorname{erf} \left(\sqrt{\pi} \frac{x-l}{r} \right) + 1 \right] \right\} \quad (5)$$

$$C_{(y)} = \frac{1}{2} \left\{ \left[\operatorname{erf} \left(\sqrt{\pi} \frac{y}{r} \right) + 1 \right] - \left[\operatorname{erf} \left(\sqrt{\pi} \frac{y-L}{r} \right) + 1 \right] \right\} \quad (6)$$

$C_{(x)}$, $C_{(y)}$, separately are called subsidence distribution coefficients of $A(x,y)$ in trend and tendency.

For flat seam, when $x = \frac{l}{2}, y = \frac{L}{2}$, the point $A(x,y)$ is center of geometry, so the equations are following:

$$C_{(x=\frac{l}{2})} = \frac{1}{2} \left\{ 2 \operatorname{erf} \left(\frac{\sqrt{\pi}}{2} \cdot \frac{l}{r} \right) \right\} = \operatorname{erf} \left(\frac{\sqrt{\pi}}{2} \cdot \frac{l}{r} \right) = n_x \quad (7)$$

$$C_{(y=\frac{L}{2})} = \frac{1}{2} \left\{ 2 \operatorname{erf} \left(\frac{\sqrt{\pi}}{2} \cdot \frac{L}{r} \right) \right\} = \operatorname{erf} \left(\frac{\sqrt{\pi}}{2} \cdot \frac{L}{r} \right) = n_y \quad (8)$$

The subsidence value of point $A(x,y)$ is max in local geological and mining conditions, the equation is following:

$$W_m = W_A = W_{\max} C_{(x_A)} C_{(y_A)} = W_{\max} n_x n_y \quad (9)$$

n_x, n_y , separately are called as coefficients of mining influence in trend and tendency, it can be used for judging whether the working face is critical mining. The coefficients of mining influence are in Table 1, when $\frac{l}{r} \geq 2.0$, coefficient of mining influence $n_x \approx 1.0$, it can be considered as $l \approx 2r$, the working face has been critical mining in this direction.

Tab. 1. Calculation of coefficients of mining influence

$\frac{l}{r}$	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
$\operatorname{erf} \left(\frac{\sqrt{\pi}}{2} \times \frac{l}{r} \right)$	0	0.198	0.383	0.547	0.684	0.789	0.867	0.920	0.955	0.976	0.988

The max subsidence value of subcritical mining is calculated by empirical equation is following:

$$W_m = W_{\max} n \quad (10)$$

$$n = 2^{-3} \sqrt[3]{n_1 n_2} \quad (11)$$

n_1, n_2 are called as coefficients of mining influence in trend and tendency, and the values are following:

$$n_1 = 0.7 \sim 0.9 \frac{l}{H_0}, n_2 = 0.7 \sim 0.9 \frac{L}{H_0} \quad (12)$$

where: H_0 – the average mining depth of working face.

Compared with other empirical equations, probability integral method is more convenient for application.

2.2. Deformation calculation model in arbitrary direction

It is assumed that the angle between the τ direction and the x axis of the calculation coordinate system is φ , then the deformation along the τ direction separately are following:

(1) Slope along the τ direction

$$\begin{aligned} i(x, y, \varphi) &= \frac{\partial W(x, y)}{\partial \tau} = \frac{\partial W(x, y)}{\partial x} \cos \varphi + \frac{\partial W(x, y)}{\partial y} \sin \varphi \\ &= \frac{1}{W_{\max}} \left[i_{(x)}^0 W_{(y)}^0 \cos \varphi + W_{(x)}^0 i_{(y)}^0 \sin \varphi \right] = i_{(x)}^0 C(y) \cos \varphi + i_{(y)}^0 C(x) \sin \varphi \end{aligned} \quad (13)$$

(2) Curvature along the τ direction

$$\begin{aligned} K(x, y, \varphi) &= \frac{\partial i(x, y, \varphi)}{\partial \tau} = \frac{\partial i(x, y, \varphi)}{\partial x} \cdot \cos \varphi + \frac{\partial i(x, y, \varphi)}{\partial y} \cdot \sin \varphi \\ &= \frac{1}{W_{\max}} \left[K_{(x)}^0 W_{(y)}^0 \cos^2 \varphi + K_{(y)}^0 W_{(x)}^0 \sin^2 \varphi + i_{(x)}^0 i_{(y)}^0 \sin 2\varphi \right] \end{aligned} \quad (14)$$

(3) Surface displacement along the τ direction

$$U(x, y, \varphi) = bri(x, y, \varphi) = \frac{1}{W_{\max}} \left[U_{(x)}^0 W_{(y)}^0 \cos \varphi + U_{(y)}^0 W_{(x)}^0 \sin \varphi \right] \quad (15)$$

(4) Surface horizontal strain along the τ direction

$$\begin{aligned} \varepsilon(x, y, \varphi) &= brK(x, y, \varphi) \\ &= \frac{1}{W_{\max}} \left\{ \varepsilon_{(x)}^0 W_{(y)}^0 \cos^2 \varphi + \varepsilon_{(y)}^0 W_{(x)}^0 \sin^2 \varphi + \left[U_{(x)}^0 i_{(y)}^0 + U_{(y)}^0 i_{(x)}^0 \right] \sin \varphi \cos \varphi \right\} \end{aligned} \quad (16)$$

2.3. Relationship between parameters of probability integral method and lithology of overlying strata

Based on a large number of observations, relationship between parameters of probability integral method and lithology of overlying strata is as Table 2, relationship between parameters of probability integral method and type of overlying strata is as Table 3.

Tab. 2. Type of overlying strata [27]

Type	Nature of overlying strata	
	Main strata	Uniaxial compressive strength [MPa]
Hard	Most of them are hard sandstone and limestone. Others are sandy shale, shale and diabase	>60
Medium hard	Most of them are medium hard sandstone, limestone and sandy shale. Others are soft gravel sandy shale and compacted marl	30~60
Soft	Most of them are hard sandy shale, shale, marlite, clay, sandy clay and loose layer in Cenozoic	<30

Tab. 3. Parameters of probability integral method and type of overlying strata [27]

Type of overlying strata	Subsidence factor (q)	Horizontal displacement factor (b)	Main influence Transference angle tangent ($\text{tg}\beta$)	Offset distance of the inflection point (S)	Maximum subsidence angle (θ)
Hard	0.27~0.54	0.2~0.3	1.2~1.91	0.31~0.43 H	$90^\circ - (0.7\sim 0.8)\alpha$
Medium hard	0.55~0.84	0.2~0.3	1.92~2.40	0.08~0.30 H	$90^\circ - (0.6\sim 0.7)\alpha$
Soft	0.85~1.00	0.2~0.3	2.41~3.54	0~0.07 H	$90^\circ - (0.5\sim 0.6)\alpha$

where: α – seam dip angle, $\alpha \leq 55^\circ$; H – mining depth.

Above prediction models have already achieved visualization and extensive application, at the same time, it has enormous economic benefits, in addition, probability integral method is compiled in monographs, professional textbooks [5-23], national technical regulations and industry standards [24-27].

3. Application development of probability integral method

3.1. Extremely subcritical mining

When the ratio of mining width to mining depth is very small, mining is called as extremely subcritical mining. Aiming at the problem of the prediction result of surface movement and deformation is large by probability integral method in extremely subcritical mining, so the method that correct the subsidence factor based on the actual observations is used.

(1) Correct the subsidence factor by mining influence

Prediction parameter is a function of mining influence coefficient. Professor Wu Kan got a empirical correction model by observations [32].

$$k_q = \frac{q_s}{q} = \begin{cases} 0.97n^2 - 0.07n + 0.39, & 0.1 < n \leq 0.83 \\ 1.0, & n > 0.83 \end{cases} \quad (17)$$

where:

- n – mining influence coefficient,
- q_s – subsidence factor of subcritical mining,
- q – subsidence factor of critical mining,
- k_q – correction coefficient of subsidence factor.

(2) Correct the subsidence factor by property of overlying strata

Professor Yang Lun got a empirical correction model by mining width, property of overlying strata and surface movement starting distance [35].

$$k_q = \frac{q_s}{q} = \begin{cases} 0, & D < L \\ 1 - \exp(-C(\frac{D-L}{H})^2), & D \geq L \end{cases} \quad (18)$$

where:

- D – mining width,
- L – starting distance of surface subsidence,
- C – coefficient of overlying strata.
- Hard overlying strata: $L = 30$ m, $C = 1$.
- Medium hard overlying strata: $L = 25$ m, $C = 2$.
- Soft overlying strata: $L = 20$ m, $C = 3$.

(3) Correct the subsidence factor by mining width and depth

According to the observations in massive mines, Professor Deng Kazhong got the change law of the subsidence coefficient and the ratio of width to depth [41].

$$q_s = a \ln(D/H) + b \quad (19)$$

where:

- D – mining width,
- H – mining depth,
- a, b – coefficient,
- a, b in partial mines are in Table 4.

In addition, there are other prediction methods, such as Boltzman function and neural network, etc. [36-41].

Tab. 4. *a, b* in partial mines

Mine	<i>a</i>	<i>b</i>	<i>D/H</i>
Fengfeng	0.242	0.736	0.2~1.4
Dongmei	0.367	0.656	0.3~1.4
Hebi	0.228	0.657	0.2~1.4
Tianba	0.305	0.622	0.3~1.4
Shuang Yashan	0.579	0.553	0.9~1.3
Jixi	0.803	0.575	1.03~1.3

4. Steep seam mining

There are many steep seams in China, after mining that, the deformations of roof and floor rock strata separately spread to the surface and influence the surface. The influence of unit mining subsidence is as Figure 2.

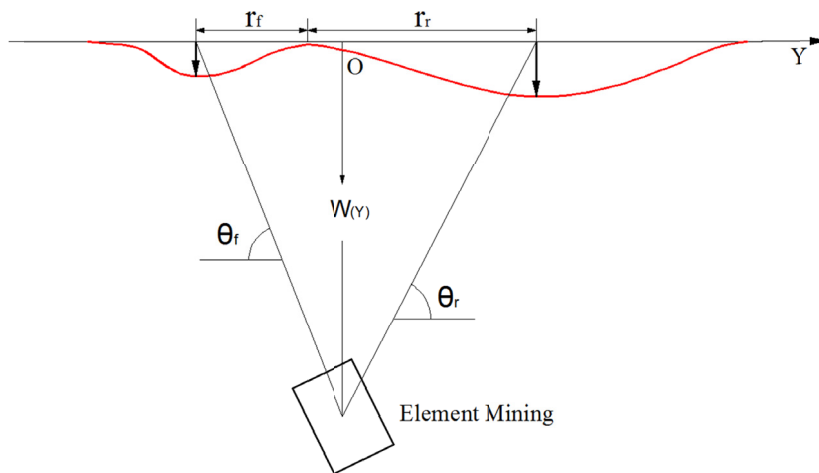


Fig. 2. The influence of element mining subsidence

Tendency direction is *y* axis, vertical direction is *z* axis, the center of the element at the surface projection point is origin, so the unit subsidence function is following [31]:

$$dW_e(y) = k_f \frac{1}{r_f} \exp\left(-\pi \frac{(y + H_0 \operatorname{ctg} \theta_j)^2}{r_f^2}\right) + (1 - k_f) \frac{1}{r_r} \exp\left(-\pi \frac{(y + H_0 \operatorname{ctg} \theta_r)^2}{r_r^2}\right) \quad (20)$$

where:

- k_f – roof influence coefficient. Range of value is [0,1],
- H_0 – unit average mining depth,
- θ_j, θ_r – maximum subsidence angle.

Comparing with flat seam mining, the law of surface movement is more complex and the prediction is more difficult in steep seam mining. Therefore, there are still many questions that should be researched.

In addition, the method is applied in metal mine, oil and gas, groundwater, halite mining and underground engineering [46-50].

6. Conclusion

- (1) Knothe influence function was introduced in 1965 and widely applied in mining subsidence prediction. At the same time, it also had enormous economic benefits, and the method had been compiled in professional textbooks, national technical regulations and technical standards.
- (2) The application of probability integral method is developed by Chinese scholars, who research the parameter correction of surface movement deformation in special mining conditions, such as extremely

subcritical mining, steep seam mining, etc. In addition, the applications of probability integral method are in metal mines, oil and gas, groundwater, halite mining and other underground engineering.

- (3) Although probability integral method did not answer the mechanism of surface subsidence, it is necessary to apply by prediction of surface subsidence, and this method still is not replaced in the near future.

This thesis commemorates the innovative contribution of Professor Knothe, meanwhile, that pays tribute to the contributions made by scholars from all countries in the development of the mining subsidence prediction.

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