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Research Progress on Prediction of Mining Subsidence in China

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Abstract

The study of mining subsidence in China started in 1950s, which learned from the successful experience of the former Soviet Union, Poland, Germany and other countries. Over the past 60 years, remarkable progress has been made in subsidence prediction methods and control techniques. This paper introduces Chinese research situation of subsidence prediction methods, models, parameters and so on. The research methods of subsidence prediction under special geological mining conditions such as steep seam, thick soil layer and mountainous terrain are expounded. The research progress on prediction method of movement and deformation under subcritical mining and residual displacement are introduced. The research results provide the theoretical and technical support for the successful application of coal mining under buildings, railways, and waterbodies in China.

Keywords: mining subsidence; prediction method; research progress; special geological and mining conditions

1. Introduction

1.1. Research of subsidence prediction methods in China

The study of mining subsidence in China started in 1950s, which learned from the successful experience of the former Soviet Union, Poland, Germany and other countries. Over the past 60 years, remarkable progress has been made in subsidence prediction methods and control techniques [1-8]. In engineering applications, there are three kinds of subsidence prediction methods in China: Typical Curve Method, Negative Exponential Function Method and Probability Integration Method. With the development of computer technology, the advantage of Probability Integration Method has been demonstrated. This method has become the main prediction method currently. The coal mines in China are widely distributed, the geological mining conditions are complex and uncertain, and the probability integration model has its limitations, which cannot be fully adapted to these complex conditions, especially for the special conditions, such as thick-steep dip angle of coal seams, thick soil layer and mountain terrain. Therefore, this paper introduces the research progress of subsidence prediction methods, models, algorithms and parameters in China.

In addition to these methods, the finite element, discrete element, boundary element, finite difference method and other numerical simulation methods are often used in the calculation of rock and ground movement.

The ground movement of strip mining and filling mining is expected to be estimated by modifying the estimated parameters or using the equivalent thickness mining method.

In addition, China has established observation stations of ground movement in dozens of mining areas, and analyzed and determined the parameters of strata and ground movement in each mining area. In particular, a probability integral method is established, which provides a basis for prediction of ground movement.

1.2. Typical Curve Method

The Typical Curve Method [2,4] is based on the measured data, viz. the dimensionless curve (called typical curve) is established reflecting the law of ground movement under certain geological and mining conditions. According to the distribution of dimensionless curve, the prediction of ground movement and deformation under certain mining conditions of similar geology is carried out. In general, typical subsidence curves of half basin in strike, dip and rise direction can be established according to observation profiles. Then the typical curves of tilt, curvature, horizontal movement and horizontal deformation are established according to the relation between movement and deformation.

The steps to set up a typical curve:

(1) According to the geological and mining conditions of a coal mine area, the observation stations are divided into several groups, and the geological and mining conditions of the observation stations in each group should be the same basically.

(2) Dimensionless transformation of measured displacement curves in observational stations.
Dimensionless method: common characteristic points of subsidence curve (such as basin boundary point or maximum subsidence point) are chosen as coordinate origin. The coordinate value of the x axis (along the strike main section) or the coordinate value of the y axis (along the inclined main section) are divided by the average mining depth H_0 or the corresponding length of half subsidence basin as dimensionless transverse axis. The displacement and deformation values are divided by the corresponding maximum values as the dimensionless longitudinal axis, and then the measured displacement and deformation curves are transformed into dimensionless curves.

(3) By comparing the dimensionless curves of movement and deformation with the similarities in each group, the average curves are obtained, which are the typical curves of the movement and deformation distribution of each group.

1.3. Negative exponential Function Method

The Negative Exponential Function Method, Weibull distribution function method belong to the class of section function method. The Negative Exponential Function Method [2] is a method for describing the ground subsidence profile by using a negative exponential function. It can be used to predict the ground movement and deformation caused by underground mining of a rectangular coal panel.

When mined area reached full mining, the surface movements and deformations, along with the strike of the main section of the half-basin, can be predicted by follows:

$$\begin{cases} W(x) = W_0 \cdot e^{-a(c-\frac{x}{H})^n} \\ i(x) = \frac{W_0}{H} \cdot a \cdot n \cdot (c - \frac{x}{H})^{n-1} \cdot e^{-a(c-\frac{x}{H})^n} \\ k(x) = \frac{W_0}{H^2} \cdot a \cdot n \cdot (c - \frac{x}{H})^{n-2} \cdot [a \cdot n \cdot (c - \frac{x}{H})^n - n + 1] \cdot e^{-a(c-\frac{x}{H})^n} \\ u(x) = B \cdot i(x) \\ \varepsilon(x) = B \cdot k(x) \end{cases} \tag{1}$$

where:

W_0 – the maximum subsidence, mm,
a – the transverse development coefficient,
c – the position coefficient,
n – the shape coefficient,
B – can be obtained by the following formulas, mm.

$$\begin{cases} B = \frac{b \cdot d}{0.413n - 0.213} \\ d = H \cdot \sqrt[n]{\frac{n-1}{a \cdot n}} \end{cases} \tag{2}$$

where: b – the coefficient of horizontal movement.

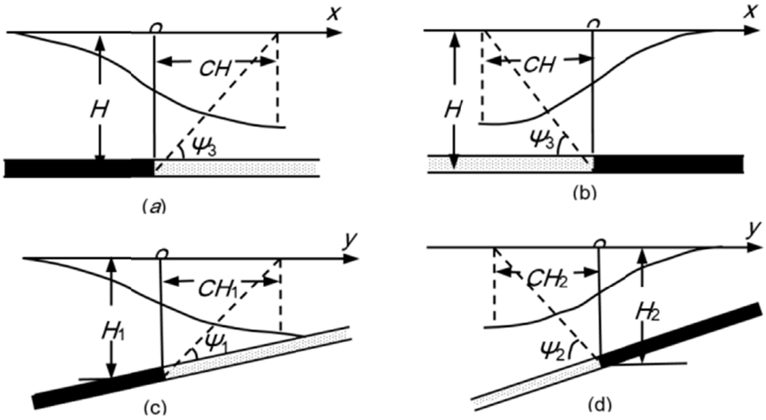

Fig. 1. Coordinate system along main section when reached to critical mining [2]
(a) – pointing to the gob along the x-axis of the strike main section, (b) – pointing to the pillar along the x-axis of the strike main section, (c) – pointing to the gob along the y-axis of the inclined main section, (d) – pointing to the pillar along the y-axis of the inclined main section

1.4. Formation and basic idea of Probability Integral Method

Probability Integration Method (PIM) is a prediction method established by Liu Baochen and Liao Guohua based on the stochastic medium theory proposed by J. Litwiniszyn. The basic theory and algorithm of this method can be seen in the monograph "the basic Laws of Coal Mine Surface Movement in Coal Mines" [6,7]. The background in the book is as follows:

The book "Basic laws of Surface Movement in Coal Mines" written by Liu Baochen, Liao Guohua in 1965 introduced the research work on mining subsidence of Polish scholars:

In 1950, W. Budryk and S. Knothe put forward the theory and formula for calculating the ground-movement and deformation of horizontal coal seam.

After 1954, the concept of randomness was introduced to study the laws of rock movement by J. Litwiniszyn.

The book discussed rock movement by studying the effects of unit mining. The author thinks that it is an effective theoretical method to discuss the law of rock movement from the point of view of probability theory. Based on the theory of stochastic medium, it is put forward that the variability and randomness of rock movement should be considered, in order to guide the application in production practice. So Liu began to study the theory of strata and ground movement from the point of view of mechanics. The unit subsidence basin in the book is built by probability theory, which is different from J. Litwiniszyn's in the form. The method is simpler, more intuitive and still strict. The research parameters and application of this method are convenient and effective.

The book indicated that the research work on PIM was developed on the basis of the work of S. Knothe and J. Litwiniszyn's.

Probability Integral Method [8] is based on unit mining to study the expression of subsidence basin. The basic idea is: the ground movement caused by unground mining is a random event, and the mining area can be subdivided into a finite or infinite number of micro units. Each micro unit produces a corresponding unit subsidence basin with normal distribution. The entire sinking basin of a rock or ground caused by a whole mining area, which equals to the sum of the effects of all micro units. Therefore, the whole subsidence basin can be expressed as the integral expression of probability density function.

In the model, five parameters are included: subsidence coefficient (q), horizontal displacement coefficient (b), main influence angle tangent ($\tan\beta$), mining influence propagation angle (θ), inflection point offset coefficient (k_s).

$$W_e\left(x,y\right)=\frac{1}{r}e^{-\pi\frac{x^2+y^2}{r^2}} \tag{3}$$

$$W\left(x,y\right)=W_{cm}\iint\limits_{\Omega}We\left(\zeta-x,\eta-y\right)d\zeta d\eta \tag{4}$$

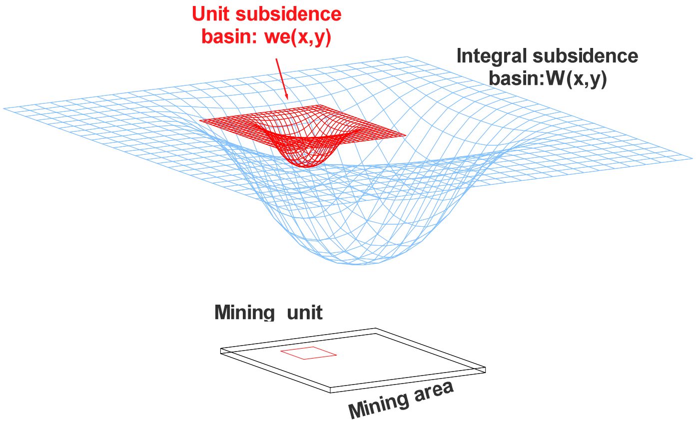

Fig. 2. Probability integral prediction diagram [9]

where:

Ω – coal seam mining area,

W_{cm} – maximum subsidence of the surface at critical mining, mm.

From the perspective of random medium theory, the random medium movement model is morphological similar to the movement of rock and soil caused by underground mining, which is the similarity of phenomena, not the mechanics. When it describes the subsidence patterns of different geological and mining conditions, there are the following limitations:

(1) The subsidence function of unit mining is based on horizontal layered ore body. For inclined and steep layered ore body, the similarity will be reduced if the ore body is treated by approximate segmenting.

(2) The unit mining subsidence function reflects the inevitable effect of the mining area (whether the size is sufficient (critical) or not) on the rock layer or the ground, but it does not reflect the effect of the change of sufficient (critical) degree caused by the mining size.

(3) The consistent propagation of the mining influence function in the strata, the propagation differences in different strata cannot be reflected.

The subsidence prediction method of special geological and mining conditions is introduced below.

2. Subsidence prediction of steep seam mining

Four kinds of subsidence prediction methods are introduced as follows: Pearson's III Formula Method, Goaf Vector Method, Vertical Integration Method, and Propagation Angle Integration Method.

2.1. Pearson's III Formula Method

Pearson III Formula Method is a section function method for predicting the ground movement of steep seams. It is suitable for the calculation of the movement and deformation of the main section of the ground subsidence basin caused by underground mining of a rectangular coal panel [1,9].

On the inclined main section of subsidence basin, under the coordinate system, which takes the movement boundary of the floor as the origin and the dip direction as the X axis (Fig. 3), The formulas for the subsidence and horizontal movement of the inclined main section are as follows:

$$W(x) = a_1 W_{\max} Z^{a_2} \exp(-a_3 Z) \tag{5}$$

$$u(x) = q(B - P(x))i(x) \tag{6}$$

where:

$$Z = \frac{x}{L_a}, L_a = \frac{H_1(\operatorname{ctg}\lambda_0 + \operatorname{ctg}\beta_0) + M}{\sin\alpha}, \text{ m}$$

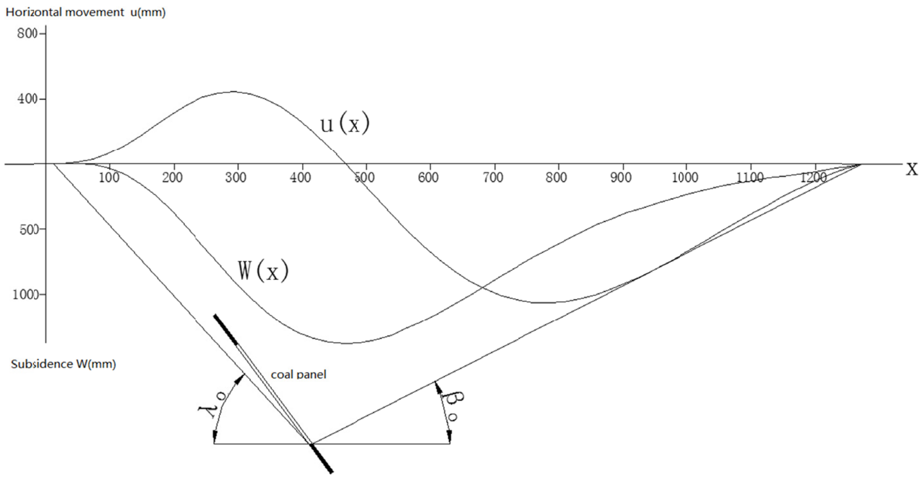

Fig. 3. Coordinate system of Pearson III Formula method [9]

$$W_{\max} = K_a \frac{\Delta H}{\sqrt{H_0}} M \cos\alpha \sqrt{n}, \text{ mm}$$

$$P(x) = B\left[1 - b_1(1-Z)^{b_2} \exp\left(-b_3(1-Z)\right)\right]$$

$$B = b_\alpha L_a$$

N – mining degree coefficient in strike direction,

$n = \sqrt{\frac{0.7l}{H_0}}$ (take $n = 1.0$ when $n > 1.0$),

α – the dip angle of coal seam,

M – the thickness of coal seam,

H_1, H_0, ΔH – mining depth of dip direction boundary, average mining depth, and vertical height of interval,

λ_0, β_0 – limit angle of floor and roof,

q – horizontal movement reduction coefficient (normally is 1.6),

K_a – subsidence influence coefficient,

b_α – horizontal movement coefficient on inclined direction,

L_a – total length of subsidence basin,

l – strike length of coal panel,

a_1, a_2, a_3 – model coefficient of subsidence basin,

b_0, b_1, b_2, b_3 – model coefficient of horizontal movement.

2.2. Goaf Vector Method

Goaf Vector Method is a mining subsidence prediction model and parameter analysis method based on variation of dip angle of coal seam [9].

The ideas of modeling are as follows: (1) According to the variation of dip angle of coal seam, the mining prediction method of steep seam can be decomposed into four processes: horizontal → inclined → steep → vertical to study, which connects and unifies the prediction methods of horizontal coal seam, inclined coal seam and steep coal seam mining. (2) Based on the theory of stochastic medium, the element mining area is regarded as vector, which is decomposed into horizontal and vertical component, then the subsidence influence function is established respectively, and the principle of equivalent effect is used to synthesize subsidence basin caused by the two kinds of components micro-element mining. The two components of the mining unit are integrated respectively in the horizontal and vertical goaf, and the calculation formula of the ground movement and deformation is established according to the superposition principle.

The principle of vector method is shown in Figure 4. In the underground mining area S, the mining thickness of coal seam is M, and the dip angle α is constant. S is divided into micro mining unit (dS) along the inclined direction of coal seam. Taking the center of the unit as the origin o, the element coordinate system (xoy) is established (x points to strike direction, y points to inclined direction). The mining unit dS is regarded as the area vector, its size is (1 × 1) and the direction of dt is the normal direction of dS. Area vector dS is decomposed into horizontal component dS_h and vertical component dSv. The size of dSh is (1 × cosα), the size of dSv is (1 × sinα). The vector form of the mining unit i.

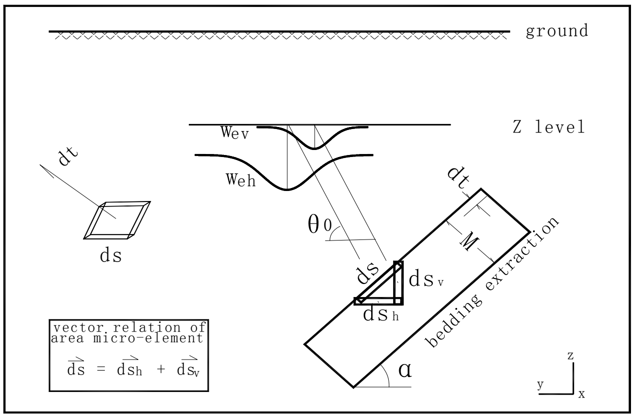

Fig. 4. The principle of Goaf Vector Method [9]

$$dS = dSh + dSv \tag{7}$$

The expressions of ground subsidence basins $W_{eh}(x,y)$, $W_{ev}(x,y)$ for horizontal component dS_h and vertical component dS_v caused by unit mining are as follows:

$$W_{eh}(x,y) = \frac{\cos\alpha}{r_h^2} e^{-\pi \frac{x^2+(y-H\operatorname{ctg}\theta_0)^2}{r_h^2}} \tag{8}$$

$$W_{ev}(x,y) = \frac{\sin\alpha}{r_v^2} e^{-\pi \frac{x^2+(y-H\operatorname{ctg}\theta_0)^2}{r_v^2}} \tag{9}$$

where:

$W_{eh}(x,y)$, $W_{ev}(x,y)$ – subsidence basins caused by dS_h and dS_v mining, mm,
r_h, r_v – main influence radius of horizontal and vertical coal seam mining, m,
H – mining depth at point P of dS, m,
θ_0 – mining influence propagation angle, °.

2.3. Vertical Integration Method

The ground subsidence basins after mining in steep seam are showed in “ladle” and “pocket” shaped subsidence basin. The ground movement and deformation of the “ladle” subsidence basin occur in the roof overburden, and the calculation method of equivalent working face can still be used. The ground movement and deformation of the “pocket” shape subsidence basin have been extended to the floor of coal seam, and the method of depth integration can be used [11,12,13].

Subsidence:

$$W(x,y) = q \cdot \iiint_G \frac{1}{r(z)^2} \cdot e^{-\pi \frac{(\eta-x)^2+(\zeta-y)^2}{r(z)^2}} \cdot d\eta \cdot d\zeta \cdot dz \tag{10}$$

where:

$r(z)$ – main influence radius at depth z, m,

G – mining space,
q – subsidence coefficient, for steep coal seam ($\alpha > 75°$), which is the volume ratio of subsidence basin to mining seam,
b – horizontal movement coefficient,
x, y – relative coordinates of calculation points (Consider inflection offset), m,
θ_0 – mining influence propagation angle, °.

2.4. Propagation Angle Integration Method – for mining of steep and extra thick coal seam

Horizontal / oblique sliced mining method is generally used in steep and ultra-thick coal seams ($M > 10$ m, $\alpha > 45°$). Its mining space is inclined to rock layer, the roof strata moves along normal direction tend to dip direction, the upper loose body collapses vertically, and the floor strata slip along tangential direction. In the inclined profile, the propagation direction of strata movement is inconsistent. Therefore, a prediction model of mining subsidence based on the variation of mining influence propagation angle is established [14].

On the inclined main section, the length of horizontal sliced coal panel is l, and the sliced coal panel is divided into mining units along the horizontal direction (Fig. 5). The influence propagation angle of roof unit is constant θ_{01}, and the floor unit is θ_{02}, the influence propagation angle of the element mining between the roof and the floor is $\theta_0(x,\alpha)$, it depends on the interface dip angle α and the horizontal position x to the middle of interface.

$$H \cot\theta_0(x) = z_1 \cot\theta_{01} + (H - z_1)\cot\theta_0(x,\alpha) \tag{11}$$

where:
H – mining depth, m,
z_1 – height of unit mining propagating to interface, m.

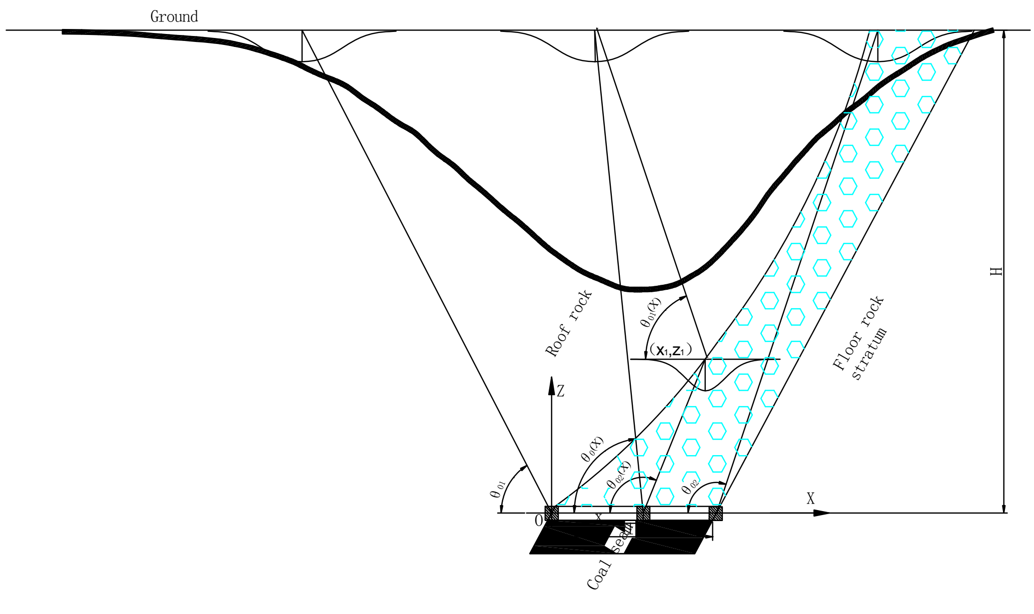

Fig. 5. Diagram of surface subsidence of unit mining based on influence propagation angle [14]

Considering the location of the unit, the direction of propagation and the complex variation of the rock strata and the unconsolidated media, it is difficult to determine the propagation angle $\theta_0(x,\alpha)$. The influence propagation angle $\theta_0(x)$ of the mining unit in the middle of the working face varies with the mining coordinate x. Assuming that the influence propagation angle of mining is linear from roof to floor, the influence propagation angle of any unit mining is as follows:

$$\theta_0(x) = \theta_{01} + \frac{x}{l}(\theta_{02} - \theta_{01}) \tag{12}$$

Or

$$\theta_0(x) = 90^{\circ} - [k_1 + \frac{x}{l}(k_2 - k_1)]\alpha \tag{13}$$

where:

- θ_{01} – mining influence propagation angle of roof unit, $\theta_{01} = 90° - k_1\alpha$, °,
- θ_{02} – mining influence propagation angle of floor unit, $\theta_{02} = 90° - k_2\alpha$, °,
- k_1, k_2 – the coefficient of mining influence propagation angle of roof and floor,
- l – the length of sliced coal panel, m,
- α – dip angle of coal seam, °,
- x – horizontal distance between mining unit and roof, m.

The expression of strata subsidence basin for unit mining is as follows

$$\begin{aligned} W_e(x,z) &= \frac{1}{r_z}\exp\left\{-\frac{\pi[x - z\cdot\cot\theta_0(x)]^2}{r_z^{\,2}}\right\} \\ &= \frac{1}{r_z}\exp\left\{-\frac{\pi[x - z\cdot\cot(90° - (k_1 + \frac{x}{l}(k_2 - k_1)\alpha))]^2}{r_z^{\,2}}\right\} \end{aligned} \tag{14}$$

where:

- $W_e(x,z)$ – the subsidence value of (x,z) point caused by unit mining, mm,
- r_z – the main influence radius of the subsidence basin at the position whose depth from working face to overlying rock is z, m.

For ground, $z = h$, $r_z = r$.

$$W_e(x) = \frac{1}{r}\exp\left\{-\frac{\pi[x - h\cdot\cot(90° - (k_1 + \frac{x}{l}(k_2 - k_1)\alpha))]^2}{r^2}\right\} \tag{15}$$

3. Prediction method of ground movement in mountain area mining

This method [10] is suitable for horizontal and gently inclined coal seams with average slope angle less than 30°.

Subsidence

$$\begin{aligned} W'(x,y) = W(x,y) + D_{x,y}\{&P[x]\cos^2\varphi + P[y]\sin^2\varphi \\ &+ P[x]P[y]\sin^2\varphi\cos^2\varphi\,\mathrm{tg}^2\alpha'_{x,y}\}W(x,y)\,\mathrm{tg}^2\alpha'_{x,y} \end{aligned} \tag{16}$$

where: $W(x,y)$ – subsidence at any point (x,y) in the flat ground under the same conditions, mm.

The main influence radius r is calculated by the following formula:

$$r(x,y) = \frac{H(x,y) - H_0}{\tan\beta} \tag{17}$$

- $H(x,y)$ – the elevation of ground predicted point (x,y) in mountain area, m,
- H_0 – the average elevation of coal panel floor, m,
- $D_{x,y}$ – ground characteristic coefficient of point (x,y),
- $\alpha'_{x,y}$ – slope angle of ground point (x,y), °,
- φ – inclined direction angle of point (x,y), °,

$\alpha'_{x,y}$, and φ are calculated from the x axis forward in the counterclockwise direction.

$P[x]$, $P[y]$ – the slip effect function of strike and inclined main section.

$P(x)$ as the formula to calculate: $P(x) = 1 + A \cdot e^{-\frac{1}{2}(\frac{x}{r}+P)^2} + W_m \cdot e^{-t(\frac{x}{r}+P)^2}$

A, P, t – slip influence parameter, its reference value $A = 2\pi$, $P = 2$, $t = \pi$.

4. Introduction on prediction method for the condition of thick unconsolidated layer

4.1. Unit subsidence model modification

In the probability integral method, the special factor of thick loose layer is not taken into account, but the action of overlying bedrock is considered. In fact, the subsidence of the ground is the result of the comprehensive action of overlying bedrock and loose layer. So in the new model, we have to consider the influence factors of the thickness of loose layer. The slow convergence at the edge of the subsidence basin is most likely related to the existence of loose layers. Therefore, it is necessary to adjust the convergence of the influence function by adjusting the main influence radius, and make it suitable for the case of thick loose layer. So the unit subsidence basin of the modified model is expressed as [16]:

$$W_e(x) = \frac{1}{r}\exp\left\{-\frac{\pi x^2}{\left[r + (H/h)^n\right]^2}\right\} \tag{18}$$

where:

H – average mining depth, m,

h – the thickness of loose layer, m,

n – the loose layer influence coefficient.

4.2. Construction of subsidence model for the loose layer

Based on measured data of Dongpang Mine, Xingtai mining area, and with the aid of simulation test of equivalent material, the rules of ground movement caused by mining under thick alluvium are described. The method for measuring the effect of mining is put forward. The problems on the larger subsidence factor and the active period of surface movement are expounded. The mechanism of displacement and failure in the thick unconsolidated soil layer and the rock is revealed. The relation between the strata movement and the alluvium subsidence is also revealed [20].

For the ultra-thick unconsolidated soil layer mining area, the characteristics of the bedrock and alluvium are very different, and cannot be regarded as the same medium. They should be treated in two different media. Therefore, it is appropriate to use double-layer medium theory to solve the spatial and temporal process of rock and ground movement caused by mining. The internal subsidence of loose layer is $W(x,y,t)$, which can be corrected by the subsidence below the bedrock surface $W_0(x,y,t)$ [21].

$$W\left(x,y,t\right) = W_0\left(x,y,t\right)\left[\exp\left(1-\frac{h_z-h}{2h}\right)\exp\left(\frac{2x-L}{2L}\right)+A\right] \tag{19}$$

where:

h_z – height of point in alluvium to the ground, m,

h – the thickness of alluvial layer, m,

L – advancing length of working face, m,

t – subsidence time, day,

$W_0(x,y,t)$ – subsidence of the bedrock surface below (x,y) points, mm,

A – constant, 0.2~0.25.

5. Subsidence prediction for different critical degree of mining

The strata and ground movement of subcritical and critical mining, there are not only great differences, but close relations. The unit mining subsidence function of the Probability Integral Method reflects the inevitable influence of the mining area (whether the size is sufficient or not) in the rock stratum or the ground, but it does not reflect the effect of the change of critical degree caused by the mining size. Therefore, which is need to be improved or modeled separately [15,16].

Utilizing the mining critical degree-ratio of width to depth of coal panel, $k_L = \dfrac{L}{H_0}$

where:

L – width of the coal panel, m,

H_0 – average mining depth of coal panel.

The subsidence characteristics of subcritical mining show that the maximum subsidence value and inflection offset of the ground are mainly affected by the critical degree of mining. Therefore, by introducing the function of ground subsidence rate and inflection point offset into the probability integral model, we can describe the subsidence state of subcritical mining.

The measured results show that there is a "slow-rapid-slow" increment relationship between the subsidence rate and the ratio of width to depth. The Boltzmann function can be used to fit (Fig. 6). The basic form of the Boltzmann function is:

$$\eta(k_L) = \mathrm{BZM}(k_L; A_1, A_2, A_3, A_4) = A_2 + \frac{A_1 - A_2}{1 + e^{(k_L - A_3)/A_4}} \tag{20}$$

$$S(k_L) = S_0\left[2\eta(k_L)\frac{k_L}{k_{LC}} - 1\right] \tag{21}$$

where:

$\eta(k_L)$ – ground subsidence ratio of subcritical mining,

$S(k_L)$ – inflection offset of subcritical mining,

S_0 – inflection offset of critical mining, m

A_1, A_2, A_3, A_4 – the coefficient of Boltzmann function, take $A_1 = 0$, $A_2 = q$

k_{L0} – ratio of width to depth at inflection point of Boltzmann curve,

k_{LC} – ratio of width to depth in critical mining.

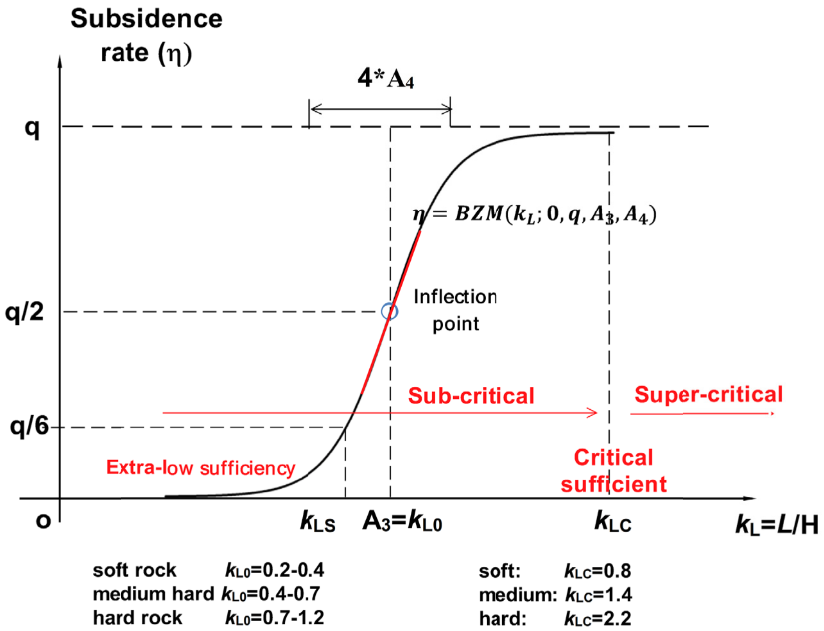

Fig. 6. Sub-critical mining subsidence rate curve simulated with Boltzmann function

According to the law of measured data, for the overlying strata of different lithology, the interval of k_{L0} is: soft rock k_{L0} = 0.2-0.4, medium hard k_{L0} = 0.4-0.7, hard rock k_{L0} = 0.7-1.2, the interval of k_{LC} is: soft rock k_{LC} = 0.8, medium hard k_{LC} = 1.4, hard rock k_{LC} = 2.2.

Thus, by introducing Boltzmann's function of mining subsidence rate and inflection offset on the ratio of width to depth, a prediction model $W(x, y)$ considering different mining subcritical degree is established.

$$W_e\left(x,y\right)=\frac{\eta\left(k_L\right)}{r^2}e^{-\pi\frac{x^2+y^2}{r^2}} \tag{22}$$

$$W\left(x,y\right)=W_{cm}\iint\limits_D\eta\left(k_L\right)W_e\left(\xi-x,\zeta-y;S\left(k_L\right)\right)d\xi d\zeta \tag{23}$$

where:

$W_e(x, y)$ – unit subsidence of points (x, y), mm,
$W(x, y)$ – integral subsidence of points (x, y), mm,
W_{cm} – maximum subsidence under critical sufficient mining, mm,
D – mining area,
r – the main influence radius, m.

6. Prediction method on residual movement and deformation

For the abandoned goaf caused by longwall mining, the deformation sources of residual movement of rock stratum and ground are mainly derived from three aspects [17-19]:

(1) Movement and deformation of overburden rock and ground caused by void compaction of fractured rock mass in goaf, which mainly occurred the fracture zone. Because of the large stress and high compaction degree in the central goaf, the voids of the goaf are small. However, due to the decrease of the force on the edge goaf, the compaction degree of the voids in the fractured rock mass is low and the volume of the voids is larger.

(2) Movement and deformation of overburden rock and ground caused by residual cavities collapse, which mainly occurred at the edge of the coal panel and the outcrop of steep coal seam. According to the borehole measurements in Xinhe and Liuxin Coal Mine, in Xuzhou city, the height of the cavities appearing at the edge of the working face varies between 0.2 and 2.0 meters, which is about 0.08 to 0.8 times of the thickness of the coal seam.

(3) The original separated layer between the strata of mining is closed by disturbance, resulting in movement and deformation of overlying rock and ground, which mainly occurred in the fractured and curved rock bodies above the caving zone, and mainly above the central part of the goaf.

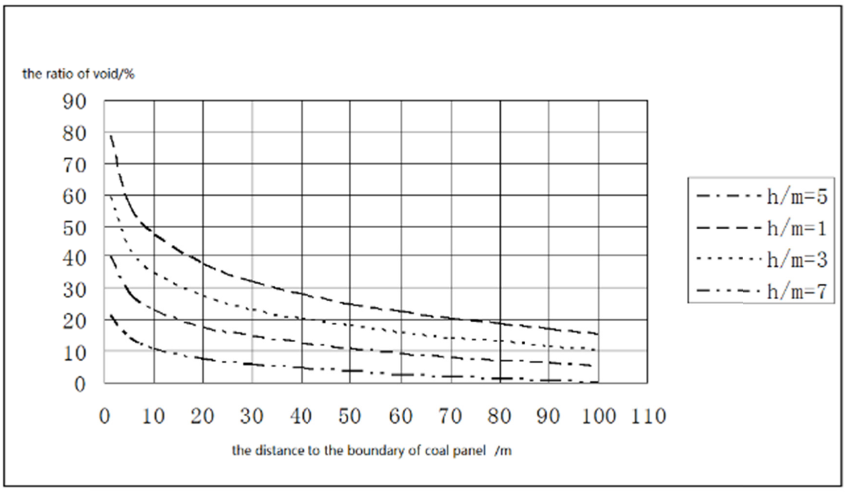

Fig. 7. Relationship between the void of fractured rock mass and the position of working face and the ratio of height to thickness [19]

Two methods can be used to predict the residual subsidence in the abandoned goaf [17,19].

(1) Based on the measured data, the relationships between the dynamic residual subsidence coefficient q_r the residual tangent of main influence angle $\tan\beta_r$ respectively and the working face stopped duration are established, then the probability integral method is used to predict the residual movement deformation at different time.

The residual subsidence coefficient q_r can be fitted by the following formula:

$$q_r = a\, e^{-bt} \tag{24}$$

Fitting parameters:

$$a = 0.0286 \ln\left(\frac{D}{H}\right) + 0.0396,\; b = -0.1195 \frac{H}{DM} + 0.4182$$

The residual tangent of main influence angle $\tan\beta_r$ can be fitted by the following formula:

$$\tan\beta_r = 1 e^{-kt} \tag{25}$$

Fitting parameters:

$$l = 2.5376 \frac{DM}{H} - 0.6766,\; k = 0.0151 \frac{DM}{H} - 0.0018$$

where:

- t – the working face stopped duration, day,
- D – the width of coal panel, m,
- H – the mining depth, m,
- M – the thickness of coal seam, m.

(2) Calculation method on the residual deformation of the abandoned goaf based on the characteristics of fractured rock mass.

According to the constitutive relation of fractured rock mass, the deformation of abandoned goaf mainly comes from the compaction of crack in caving zone, then the relation between residual deformations and dilatation coefficient of fractured rock mass, the height of collapse zone h_m, vertical compression deformation ε_m, the compressive strength of caving rock mass σ_c and overburden stress σ are established.

$$w = \frac{h_m \varepsilon_m}{10.39 \frac{\sigma_c^{1.042}}{\sigma} \left(1 - \varepsilon_m\right)^{7.7} \varepsilon_m + 1} \tag{26}$$

where:

- w – surface residual subsidence, mm,
- h_m – the height of collapse zone, m,
- ε_m – vertical compression deformation, mm/m,
- σ_c – the compressive strength of caving rock mass, MPa,
- σ – overburden stress, MPa.

7. Concluding remark

The geological conditions of coal mining in China are complicated and the subsidence problem is very complex. It is necessary to study the prediction method of mining subsidence. This paper introduces the research progress of the main methods and models of subsidence prediction in China. The research methods of subsidence prediction under special geological mining conditions such as steep seam, thick unconsolidated soil layer and mountainous terrain are expounded. The research progress of the prediction of movement deformation for subcritical mining and the prediction of residual displacement for longwall mining are also introduced. Some methods need to be supplemented and improved. Innovative models are needed to continuously improve the theory and method of subsidence prediction of underground coal mining.

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